Offline tuning mechanism of joint angular controller for lower-limb exoskeleton with adaptive biogeographical-based optimization

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Abstract: Designing an accurate controller to overcome the nonlinearity of dynamic systems is a technical matter in control engineering, particularly for tuning the parameters of the controller precisely. In this paper, a tuning mechanism for a proportional-integral-derivative (PID) controller of lower limb exoskeleton (LLE) joints by adaptive biogeographical based-optimization (ABBO) is presented. The tuning of the controller is defined as an optimization problem and solved by ABBO, which is an iterative algorithm inspired by a blending crossover operator (BLX-α). The parameters of the migration change proportionally to the growth of iteration that conveys the error to rapid convergence by narrowing the searching space. The Lyapunov stability theory is proven for LLE nonlinear dynamic systems. ABBO algorithm is compared with other conventional optimization methods in step response, which guaranteed it was not trapped in local optima and demonstrated the lowest average error and the fastest convergence rate. The tuned controller is applied in a closed-loop system to verify its performance in the prototype. The experimental results of ABBO with PID controller ascertained that the proposed tuning mechanism is applicable in the LLE gait training.

Key words: Biogeographical-based optimization, proportional-integral-derivative controller, lower limb exoskeleton, trajectory control

1. Introduction

Currently, the rehabilitation process of the lower limb is challenging because it is labor-intensive and it highly relies on the skill of physiotherapists. The exoskeleton is a wearable robot that is used to increase human strength and stamina for rehabilitation and power enhancement for different tasks such as lifting heavy objects or standing a long time [1, 2]. In rehabilitation for disabled people who are struggling with mobility issues caused by stroke, spinal cord injuries, and aging, lower limb exoskeleton (LLE) has been utilized in providing gait assistance and a fast recovering process [3–5]. In addition, LLE releases physical stress of physiotherapists for gait training as rehabilitation devices [6–8].

The challenges of dealing with the control of the exoskeleton result from its complicated structure, which makes it difficult for wearers to use it during long rehabilitation processes. Furthermore, the control system should cope with various ranges of wearer’s weight, size, and disability level. Therefore, selection of robust control methods for joint movement is essential in the development of rehabilitation LLE. Various control approaches have been carried out for LLE specialized in rehabilitation [9, 10].

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Developing control strategies in rehabilitation robots plays an important role in achieving satisfactory performance for various rehabilitation treatments [11]. However, due to the complexity of its dynamic plant with unknown uncertainties and variations, the development of a classical control system for LLE has been hindered. Therefore, the classical control system is usually combined with other techniques. For instance, He et al. [12] established a neural network (NN) feedback control system for a two-degree of freedom (DoF) knee rehabilitation robot with unknown parameters to overcome the disturbance and gain robustness of the control system. Shan et al. [13] applied a fuzzy logic controller and proportional-integral-derivative (FLC-PID) for a wearable one-DoF orthosis with active knee joint for walking assistance to reduce consumption of muscular power during gait training. Zhang et al. [14] classified the control part of the lower limb rehabilitation robot into stand and swing phases. They analyzed the kinetic model for both phases and employed sliding mode and fuzzy compensation method to reduce the disturbances in swing phase. Meanwhile, the cerebellar model articulation and proportional-integral-derivative (PID) controller for stand and swing phases have been established for stabilizing the control system. In addition, the stability validation of the proposed classified control method for a simulated model has been developed by Lyapunov stability theory. In a similar study, Li et al. [15] used a hybrid phase based on stand and swing phases of gait training to determine dynamic equations by Lagrangian and established their control system by a combination of the fuzzy and sliding mode controller. Even though there is a lot of research on the control of lower limb exoskeleton, most of the control techniques of rehabilitation robots required a treadmill, or at least one physiotherapist is needed when training. They manifest good performance of gait rehabilitation, but they are not suitable for a multiple-joint training application and they are complicated and high-cost. Hence, it is important to develop an affordable LLE with a high-accuracy control system by combining nonconventional optimization techniques with a simple classical control system such as PID.

In this paper, an adaptive biogeographical-based optimization (ABBO) with proportional migration operator parameter is presented. Different from other population-based methods, in biogeographical-based optimization (BBO), the qualities of bad solutions can be improved by accepting new features from good ones due to the ability to generate competitive candidates compared with other metaheuristic algorithms. BBO was introduced by Simon [16] as a novel optimization algorithm inspired by the natural biogeographical phenomenon, which has been used in many similar problems that other algorithms such as genetic algorithm (GA), particle swarm optimization (PSO), and ant colony optimization (ACO) are applicable [17, 18]. However, the uniqueness of BBO is in its migration operator, which is used for generating the new population from previous populations, and the mutation operator that employs a diversity of the possible solutions. Many types of research have been accomplished to increase its performance by modification of its operators. For instance, Wen et al. [19] analyzed the characteristics of global topology and direct-copying migration strategy to increase the convergence speed and accuracy and avoid homogenization of habitats in comparison with conventional BBO. In another work, to deal with reliability redundancy allocation problems of four various case studies such as series, series-parallel, bridge, and over-speed protection systems, Garg [20] presented an effective penalty-guided BBO algorithm for dealing with constraints and penalizing the infeasible solution. On the other hand, Reihanian et al. [21] developed an algorithm of BBO with two-phase migration operator to provide a balance between exploration and exploitation to decrease the chance of trapping in local optima. Statistical validation showed the superiority of their proposed algorithm to the other evolutionary algorithms.

For increasing the accuracy and robustness of the classical control system, one of the techniques is to combine it with optimization methods. So far various techniques of the optimized-based controller have
been designed by previous researchers. For instance, Misaghi et al. [22] utilized an improved invasive weed optimization algorithm to set the parameters of a PID controller for a DC motor. They defined the operators of the optimization algorithm to select a large value in focusing on more exploration at the beginning of the process. In the end, after approaching the neighbor of global optima, the searching space has been narrowed. Thus, exploration and exploitation were balanced. Pal et al. [23] employed PSO to design robust stable quadratic-optimal fuzzy controllers to achieve both robust stability and desired transient response. Their proposed method has better performance than the hybrid Taguchi genetic algorithm-based approach by achieving robust stability and desired transient response. Wang et al. [24] presented an optimal adaptive control scheme for nonlinear systems with input time-delays by using the online policy iteration algorithm. They validated their results on two simulations nonlinear examples. Therefore, although these are optimized-based controllers for the implementation of adaptive algorithms, we need to eliminate the external disturbances, which represent influences of interaction with human users during optimization process. Table 1 illustrates a literature analysis on the methods used in the design and control of LLE.

Table 1. Literature analysis of LLE control methods.

<table>
<thead>
<tr>
<th>No.</th>
<th>Reference</th>
<th>Year</th>
<th>DoF</th>
<th>Controller type</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>[26]</td>
<td>2021</td>
<td>8</td>
<td>Adaptive Lyapunov-based</td>
</tr>
<tr>
<td>5</td>
<td>[27]</td>
<td>2021</td>
<td>2</td>
<td>Reduced adaptive fuzzy</td>
</tr>
<tr>
<td>6</td>
<td>[12]</td>
<td>2015</td>
<td>2</td>
<td>Adaptive controller</td>
</tr>
<tr>
<td>7</td>
<td>[13]</td>
<td>2016</td>
<td>1</td>
<td>FLC-PID</td>
</tr>
<tr>
<td>9</td>
<td>[28]</td>
<td>2017</td>
<td>2</td>
<td>PSO-PID</td>
</tr>
<tr>
<td>10</td>
<td>[29]</td>
<td>2021</td>
<td>4</td>
<td>Dragon fly algorithm (DFA) and FLC-PID</td>
</tr>
</tbody>
</table>

In summary, the contributions of the paper are,

- An optimal PID controller for LLE rehabilitation application is presented;
- The stability of the control scheme is proven by Lyapunov approach;
- The applicability of the proposed optimization technique to minimize the steady-state trajectory error is investigated and compared with conventional optimization approaches;
- The validated results in the LLE prototype reveal efficiency of the proposed method.

The rest of the paper is structured as follows. In the beginning, the dynamic model of the LLE based on its structure is determined. The simulated model is used in a closed-loop control system with a PID controller. In the control system, ABBO is employed to tune the parameters of the PID controller by minimizing the root mean square error. Furthermore, the equation of the migration operator and its algorithm, which is an adaptation of the BBO, is explained. The performance of ABBO is compared with those of other conventional BBO algorithms. The stability of the nonlinear dynamic system of LLE is proved by the Lyapunov function. Finally, the tuned optimized control system is verified in a prototype of the LLE.
2. Dynamic model

In this paper, an LLE with a 4-DoF linkage that consists of two joints in each leg has been used. A 12V DC motor is employed and controlled by motor drivers to regulate its voltage and direction. A quadrature encoder is located at the connecting shaft to record the angle of each joint. An Arduino Mega 2560 is utilized to control the DC motor and capture the data from the encoder. Figure 1 illustrates the free body diagram of one leg of the LLE, length, and center of gravity (CoG) of each link joint. O₁ and O₂ are active hip and knee joints and O₃ represents a passive ankle joint. Femur is from O₁ to O₂ and tibia is from O₂ to the foot.

![Free body diagram of the LLE.](image)

The dynamic equation of LLE in state space is given as follows [31]:

\[
\tau = M(\theta)\ddot{\theta} + V(\dot{\theta}, \theta) + G(\theta)
\]  

where \(\tau \in \mathbb{R}^4\) is torque vector, \(\ddot{\theta} \in \mathbb{R}^4\) denotes angular acceleration vector, \(M(\theta) \in \mathbb{R}^4\) is inertia and mass matrix, \(V(\dot{\theta}, \theta) \in \mathbb{R}^4\) represents vector of centrifugal and Coriolis vector, and \(G(\theta) \in \mathbb{R}^4\) introduces gravitational vector. In this paper, Lagrangian method has been applied to determine the dynamic equation as follows:

\[
L = E_k - E_p
\]

where \(E_k\) and \(E_k\) represent total kinematic and potential energy of the links, respectively. The following equations express the torque for hip and knee while \(j = 1, 2, 3, 4\) [32, 33].

\[
\tau_j = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_j} \right) - \left( \frac{\partial L}{\partial \theta_j} \right) + B\dot{\theta}_j
\]
where $B$ denotes the friction coefficient. $\theta_j$ and $\dot{\theta}_j$ are the angular trajectory and velocity of each link, respectively. $E_k$ and $E_p$ are expressed as follows:

$$E_p = \sum_{j=1}^{4} m_j g y_j$$

$$E_k = \sum_{j=1}^{4} \left[ \frac{1}{2} m_j (\dot{x}_j^2 + \dot{y}_j^2) + \frac{1}{2} I_j \dot{\theta}_j^2 \right]$$

where $m_j$ and $I_j$ are mass and inertia of the links. $g$ is the gravity acceleration whereby $x_i$ and $y_i$ are the position of CoG for each link, respectively, given as follows:

$$x = \sum_{j=1}^{i-1} l_j \sin(\theta_j) + d_i \sin(\theta_j)$$

$$y = \sum_{j=1}^{i-1} -l_j \cos(\theta_j) - d_i \cos(\theta_j)$$

where $l_j$ and $d_i$ are length of each link and CoG, respectively. From mechanical structure in Figure 1, the torque applied to DC motor is represented as follows,

$$\tau_j = K_g \tau_m$$

where $K_g$ is the gear ratio.

3. Motor model

The torque applied to each joint is provided by a DC motor which turns electrical to mechanical energy [34]. From Kirchhoff’s law, the DC motor can be expressed as follows:

$$U - U_b = L_k \frac{dI}{dt} + RI$$

where $L_k$ and $R$ are motor coil inductance, and resistance; $U$ and $I$ are input voltage and current of the DC motor, respectively. $U_b$ represents back electromotive force voltage, which occurs due to a change in current across the coil of the DC motor and causes a change in the magnetic field and therefore produces a self-induced voltage [35, 36] as follows:

$$U_b = K_c \dot{\theta}$$

where $K_c$ is the voltage constant and $\dot{\theta}$ is angular velocity of the rotor shaft of the DC motor. The torque produced is expressed as follows:

$$\tau_m = K_m I$$

where $K_m$ is the torque sensitivity. Table 2 represents the parameters of physical features of the LLE [37]. Since the LLE has symmetric parallel legs, in Table 2 we consider physical features for one leg.
Table 2. Parameters of physical features.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>j=1</th>
<th>j=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_j$ ($kg$)</td>
<td>0.387</td>
<td>0.4</td>
</tr>
<tr>
<td>$l_j$ ($m$)</td>
<td>0.41</td>
<td>0.43</td>
</tr>
<tr>
<td>$d_j$ ($m$)</td>
<td>0.2</td>
<td>0.21</td>
</tr>
<tr>
<td>$I_j$ ($kg \cdot m^2$)</td>
<td>0.0055</td>
<td>0.0064</td>
</tr>
<tr>
<td>$K_m$ ($Nm/A$)</td>
<td>1.92</td>
<td>1.92</td>
</tr>
<tr>
<td>$R$ ($\Omega$)</td>
<td>3.7</td>
<td>3.7</td>
</tr>
<tr>
<td>$L$ ($H$)</td>
<td>1.3720</td>
<td>1.3720</td>
</tr>
<tr>
<td>$K_c$ ($V/rad.s^{-1}$)</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td>$K_g$</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$B$</td>
<td>0.035</td>
<td>0.035</td>
</tr>
<tr>
<td>$\tau_{max}$ ($Nm$)</td>
<td>9.6</td>
<td>9.6</td>
</tr>
</tbody>
</table>

$\tau_{max}$ denotes maximum torque of the DC motor. By using the parameters of Table 2, a simulation model to verify its response has been conducted as in Figure 2, in which the actual angular trajectory is captured by the encoder of the LLE. Various inputs of the hip are set as 6 V and 8 V for validation purpose. Similarly, knee 4 V and 6 V are applied as the step response in an open-loop.

![Figure 2](image)

**Figure 2.** Validation of angular trajectory in open-loop.

In Figure 2, A- and M- are the angular trajectories for actual and simulation of each joint. For instance, in Figure 2b, A-4v and M-4v are the angular trajectories while the 4v is applied to actual and simulation of knee, respectively. Figure 3 represents the error between actual and simulation for hip and knee. Figure 4 shows the torque applied to hip and knee by actuators. As can be seen it does not exceed the $\tau_{max}$ ($Nm$) which is 9.6 ($Nm$). Figure 5 illustrates the angular velocity for hip and knee for actual trajectories.

A statistical analysis is expressed in Table 3, in which ME, AE, and RSME represent maximum error, average error, and root mean square error of the angular trajectory, respectively. The units of the errors are measured in the radian.
4. Development of control strategy

In this paper, a tuning mechanism for a PID-based controller with ABBO algorithm is used to control LLE joints’ position in offline mode. PID has been regarded as one of the most popular controllers in the industry because of its ease of implementation, and efficient performance [38, 39]. The controller has a first-order transfer function in the derivative expression to maintain the stability in high-frequency noises that are produced by the
Table 3. Trajectory error for each joint in open-loop.

<table>
<thead>
<tr>
<th>Voltages</th>
<th>ME</th>
<th>AE</th>
<th>RSME</th>
<th>Voltages</th>
<th>ME</th>
<th>AE</th>
<th>RSME</th>
</tr>
</thead>
<tbody>
<tr>
<td>6V</td>
<td>0.15</td>
<td>0.022</td>
<td>0.028</td>
<td>4V</td>
<td>0.083</td>
<td>0.005</td>
<td>0.0015</td>
</tr>
<tr>
<td>8V</td>
<td>0.10</td>
<td>0.009</td>
<td>0.018</td>
<td>6V</td>
<td>0.1</td>
<td>0.006</td>
<td>0.018</td>
</tr>
</tbody>
</table>

encoder [40]. The expression of the PID controller is given as follows,

\[ C(s) = K_p + \frac{K_i}{s} + \frac{K_ds}{s + N} \] (12)

where \( N \) is the parameter of the transfer function of filter for derivative part of PID controller. \( C(s) \) exhibits a filtered PID, which is demonstrated in Figure 6.

![Filtered PID controller C(s).](image)

The gains of \( K_p \), \( K_i \), and \( K_d \) represent proportional, integral, and derivative parameters of the controller, respectively. Input \( U \) is fed to the LLE plant, and \( e \) represents the steady-state error, that is the difference between actual and desired angular trajectories of hip and knee as:

\[ e = \theta_d - \theta_a \] (13)

where \( \theta_d \) and \( \theta_a \) are the desired and actual angular trajectories, respectively.

5. Optimal controller tuning

The PID controller is tuned as an optimization problem to minimize the RMSE of joints’ trajectory. There are few functions for qualifying the cumulative summation of the error that can be used as an objective function such as integral absolute error (IAE) and integral time-weighted absolute error. In this paper, RMSE is utilized as an objective function, because it shows the calculated trajectory of each habitat spared out from the desired
input of the control system.

\[ RMSE = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n}} \]  

(14)

where \( e_i \in \mathbb{R}^n \) is an error vector, represented in Equation 13. The design variables of the optimization problem are optimized so that the objective function can be minimized \([41]\). The initial population of ABBO is set randomly. Each population consists of habitats including the parameters of the filtered PID controller, given as follows:

\[ x = [K_p, K_i, K_d, N] \]  

(15)

where \( x \) represents a habitat of a population. The size of the population is usually selected based on the decision of the designer. Although some methods have been used for selecting the number of population in evolutionary algorithms such as Taguchi which is a kind of experimental design method \([42, 43]\), population size is set as 40 based on trial-and-error. Subsequently, the habitat is evaluated by determining the objective function and sorted in ascending order. Twenty percent of the habitats remain unchanged as the elites. The rest of the habitats are changed by migration and mutation to keep the diversity which finally increases the chance for ABBO in finding global optima and avoiding trapping in the local ones. Figure 7 represents changes in emigration and immigration rate in one habitat.

![Figure 7. Changes in emigration and immigration rates.](image)

In Figure 7, \( \lambda \) and \( \mu \) are immigration and emigration rates that are set for each individual of habitats. The first habitat has the lowest emigration and the highest immigration rate. Ironically, the rates of emigration and immigration are the highest and lowest in the last habitat, respectively. For applying the migration operator, a random value has been generated to be compared with \( \lambda \) of all the habitats. If the random number is less than the \( \lambda \), the new habitats for the next iteration will be determined based on the migration operator which is inspired from blending crossover (BLX-\( \alpha \)) \([44, 45]\). The usage of blending crossover as a migration operator raises the convergence speed and the performance of the conventional BBO method. The migration operator is given as follows:

\[ x_{i,j} = \omega ((x_{k,j} + \alpha_1) - (x_{i-1,j} - \alpha_2)) \]  

(16)

where \( i \) is the number of current habitats and \( j \) represents the iteration. \( \omega, \alpha_1, \) and \( \alpha_2 \) are the parameters of
the migration operator. $k$ is selected by the roulette wheel scheme, which is based on emigration probabilities,

$$e_{p_i} = \frac{\mu_i}{\sum_{i=1}^{i_{\text{max}}} \mu_i} \quad \text{for} \quad i = 1 : i_{\text{max}}$$

(17)

where $i_{\text{max}}$ is the total number of habitats. The cumulative summation of the $e_p$ is given as:

$$C_i = \sum_{i=1}^{i_{\text{max}}} e_{p_i}$$

(18)

where $C_i$ and $e_{p_i}$ are $1 \times i_{\text{max}}$ vectors. The roulette wheel scheme selects a random value and compares it with $C_i$. The number of the first element of $C_i$ vector that is greater than the random value is selected as the $k-th$ habitat. It provides more chances for the selection of the habitats with a higher rate of emigration.

The parameters of the migration operator such as $\omega$, $\alpha_1$, and $\alpha_2$ are not constant during the iterations and are changed by increasing their number. This increases exploration of searching space at the beginning of the algorithm, and narrows to the global optima by increasing the number of iterations given as follows:

$$\omega = \omega_{\text{int}} b$$

(19)

$$\alpha_1 = 2.3b + 2.6$$

(20)

$$\alpha_2 = 2.3a + 0.1$$

(21)

where $b$ and $a$ are decreasing and increasing values between 0 and 1 as follows:

$$a = \frac{j}{j_{\text{max}}} \quad \text{for} \quad j = 1 : j_{\text{max}}$$

(22)

$$b = 1 - a$$

(23)

where $j_{\text{max}}$ is the maximum value of the iteration. The value of $\omega$ starts from 1.2 and decreases gradually. This provides the wide searching space for ABBO in the initial iterations and narrows the exploration for finding global optima at the end of the algorithm. In equations (20) and (21), the summation of $\alpha_1$, and $\alpha_2$ is 5, while $\alpha_1$ and $\alpha_2$ are downward and upward entire iterations, respectively [39, 46]. Figure 8 exhibits how migration parameters modify by changing the number of iterations.

After applying the migration operator, the mutation is carried out to keep the diversity in ABBO. The mutation probability $m_p$ is a positive number between zero and one.

$$m_p \in [0, 1]$$

(24)

The mutation operator is expressed as follows:

$$\text{if} \quad r < m_p : \quad x_{i,j} = x_{i-1,j} + \sigma \quad \text{for} \quad i = 1 : i_{\text{max}}$$

(25)

where $r$ and $\sigma$ are random values between 0 and 1. After establishing the migration and mutation operators, each habitat is evaluated by the objective function. All the habitats are sorted in descending order from the
lowest to the highest quantity. Therefore, the first habitat is the output of the ABBO. Algorithm 1 exhibits the pseudocode and flow chart.

**Algorithm 1 Pseudocode of ABBO.**

1: Start
2: Initialize 40 habitats randomly;
3: Set parameters for \( \mu \) and \( \lambda \);
4: while Number of \( i_{max} = 400 \) do;
5: for \( j=1:j_{max} \) do;
6: for \( i=1:i_{max} \) do;
7: if \( r < \lambda_i \)
8: Calculate \( x_{i,j} = \omega((x_{k,j} + \alpha_1) - (x_{i-1,j} - \alpha_2)) \)
9: end
10: end
11: for \( i=1:i_{max} \) do;
12: if \( r < m_p \)
13: Calculate \( x_{i,j} = x_{i-1,j} + \sigma \) for \( i = 1 : i_{max} \)
14: end
15: Evaluate the habitats
16: Sort the habitats
17: end while
18: Select the first habitat as a result
19: End;

In addition, a penalty function is set for each habitat, which contains the negative value equal to zero because parameters of filtered PID controller are defined as nonnegative values.

\[
x_{i,j} = \max(x_{i,j}, 0)
\]  \hspace{1cm} (26)

Moreover, in order to simulate the actual conditions for the ABBO algorithm to tune the filtered PID controller, the high-frequency disturbance is implemented in the closed-loop control system for evaluation. The
simulated disturbance $D(s)$ represents the high-frequency noises that are produced by the encoder in the actual LLE. Figure 9 shows the block diagram of the ABBO and filtered PID controller in the presence of the noises.

![Figure 9. Block diagram of ABBO and closed-loop control system.](image)

### 6. Stability analysis

**Theorem 1.** Consider a general nonlinear dynamic system

$$\dot{x} = f(x), \quad x \in \mathbb{R}^n, \quad f(0) = 0$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is a Lipschitz function. Consider a function $V : \mathbb{R}^n \to [0, \infty]$ which is positive definite and $C^2$ with locally Lipschitz gradient, which is denoted by $V_x$. Lyapunov function for the nonlinear dynamic system exists by applying the following condition:

$$\dot{V} := V_x(x) \cdot f(x) < 0 \quad \forall x \neq 0$$

The nonlinear dynamic system can be shown to be asymptotically stable if such a Lyapunov function exists [47–49]. In this paper, the closed-loop system of each joint is analyzed by Lyapunov stability theory. The transfer function of closed-loop control system is represented as follows:

$$G(s)_{cl} = \frac{\theta_a}{\theta_d} = \frac{G_i(s)C(s)}{1 + G_i(s)C(s)}$$

In the closed-loop control system, $G_i(s)$ and $C(s)$ are the third- and second-order transfer functions, respectively. Therefore, closed-loop transfer function of the LLE is shown as follows:

$$G(s)_{cl} = \frac{a_2 s^2 + a_1 s^1 + a_0}{s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s^1 + b_0}$$

The steady-state of the control system is represented as follows:

$$\dot{X} = AX + Bu$$

(31)
\[ y = CX + Du \] (32)

The matrices \( A, B, C, \) and \( D \) are represented as follows:

\[
A = \begin{bmatrix}
-b_4 & -b_3 & -b_2 & -b_1 & -b_0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\] \( B = \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} \) (33)

\[
C = \begin{bmatrix} a_2 & a_1 & a_0 \end{bmatrix} \quad D = [0] \] (34)

\( u \) is the output of the PID and \( e \) is the error of the control system shown as follows:

\[ e = r - y = r - CX \] (35)

\( y \) is the output of the plant. By defining \( O = \begin{bmatrix} 1 & \frac{1}{s} & \frac{s}{s+N} \end{bmatrix} \) and \( \theta = \begin{bmatrix} K_p \\ K_i \\ K_d \end{bmatrix} \), the PID controller can be given as follows:

\[ u = \begin{bmatrix} 1 & \frac{1}{s} & \frac{s}{s+N} \end{bmatrix} \begin{bmatrix} K_p \\ K_i \\ K_d \end{bmatrix} \left( r - CX \right) = O\theta(r - CX) \] (36)

\( r \) is the input of the control system. A Lyapunov function candidate is selected as \( V(x) = X^T PX \), where \( P \in \mathbb{R}^{4 \times 4} \) is an identity matrix, which is positive definite; therefore, \( V \) is positive as well [50]. The derivative of the Lyapunov function is shown as follows:

\[ \dot{V}(x) = X^T (A^T P + PA)X + B^T uPX + X^T PBu \] (37)

By substituting Equation 36 into Equation 37, derivative of the Lyapunov function is written as the following equation,

\[ \dot{V}(x) = X^T (A^T P + PA)X + B^T O\theta(r - CX)PX + X^T PB\theta(r - CX) \] (38)

By applying the optimization operator, the output of the control system \( y = CX \) will be equal to the input of the control system \( r \), \( (r \approx CX) \). Therefore, Equation 38 is written as follows:

\[ \dot{V}(x) = X^T (A^T P + PA)X \] (39)

where \( Q \) is substituted by \( A^T P + PA \),

\[ \dot{V}(x) = X^T QX \] (40)

If \( P \) and \( Q \) become positive and negative definite, respectively, \( \dot{V}(x) \) and \( V(x) \) will be negative and positive definite. Therefore, the nonlinear dynamic system of LLE satisfies Theorem 1.
Table 4. Parameters used in GA, PSO, and ABBO.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>GA</th>
<th>PSO</th>
<th>ABBO</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of iteration</td>
<td>400</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>No. of population</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Objective function</td>
<td>RMSE</td>
<td>RMSE</td>
<td>RMSE</td>
</tr>
<tr>
<td>No. of design variables</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Mutation probability</td>
<td>0.1</td>
<td>N/A</td>
<td>0.1</td>
</tr>
<tr>
<td>Crossover probability</td>
<td>0.1</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Inertia parameter</td>
<td>N/A</td>
<td>1.2</td>
<td>N/A</td>
</tr>
<tr>
<td>Cognitive parameter</td>
<td>N/A</td>
<td>2</td>
<td>N/A</td>
</tr>
<tr>
<td>Social parameter</td>
<td>N/A</td>
<td>2</td>
<td>N/A</td>
</tr>
</tbody>
</table>

7. Results and discussion

In order to verify the performance of the proposed ABBO, the PID controller is tuned using GA and PSO for the hip and knee joints. Table 4 shows the parameters used in GA, PSO, and ABBO.

Figures 10 and 11 compare the step response and angular velocity of PID control system tuned by GA, PSO, and ABBO. Figure 12 shows torque of the step response for hip and knee.

![Figure 10](image-url)  
(a) Hip  
(b) Knee  
Figure 10. Step response tuned by GA, PSO, and ABBO.

![Figure 11](image-url)  
(a) Hip  
(b) Knee  
Figure 11. Angular velocity of step response tuned by GA, PSO, and ABBO.
In Figure 12, the maximum torque applied is 8.2 Nm and 9.1 Nm for hip and knee, respectively, which are less than the $\tau_{max}$. Settling times of the GA, PSO, and ABBO for the hip are 1.6, 1.16, and 1 second, respectively. Similarly, the settling time for the knee is 1.5, 1.2, and 0.2 s. The AE for the hip is measured as 0.026, 0.022, 0.015 radian for GA, PSO, and ABBO, respectively. Similarly, the AE for the knee is calculated by 0.028, 0.018, 0.012 radians, respectively. These statistical data represent that ABBO performed better with the lowest average error and settling time than GA and PSO. Table 5 shows the statistical analysis for step responses of GA, PSO, and ABBO.

Table 5. Statistical analysis for step responses.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Hip</th>
<th>Knee</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Settling time</td>
<td>Overshoot</td>
</tr>
<tr>
<td>GA</td>
<td>1.6s</td>
<td>49.04</td>
</tr>
<tr>
<td>PSO</td>
<td>1.16s</td>
<td>16.25</td>
</tr>
<tr>
<td>ABBO</td>
<td>1s</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Overshoot for ABBO is the lowest. For instance, it is 91% and 73% less than PSO and GA. In the experimental test, two other types of BBO algorithms with different migration operators have been developed to compare the performance of tuning of PID ABBO. Table 6 expresses the equations of three different migration operators of the BBO that are used for tuning the controller. Constant values are used for migration operators of BBO-C and BBO-C-$\alpha$ that are inspired by arithmetic and $BLX-\alpha$ crossovers, respectively. The condition of verification and mutation operator is similar in all algorithms.

Table 6. Different types of BBO algorithm.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Migration operator</th>
<th>$\alpha_1$</th>
<th>$\alpha_1$</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBO-C</td>
<td>$x_{i-1,j} + \alpha_1 (x_{k,j} - x_{i-1,j})$</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BBO-C-$\alpha$</td>
<td>$\omega ((x_{k,j} + \alpha_1) - (x_{i-1,j} - \alpha_1))$</td>
<td>0.1</td>
<td>-</td>
<td>1.2</td>
</tr>
<tr>
<td>ABBO</td>
<td>$\omega ((x_{k,j} + \alpha_1) - (x_{i-1,j} - \alpha_2))$</td>
<td>2.3$b + 2.6$</td>
<td>2.3$a + 0.1$</td>
<td>1.2$w$</td>
</tr>
</tbody>
</table>

To compare the performances of the different BBO algorithms given in Table 6, the three algorithms are applied to the closed-loop control system. The optimization algorithms are run ten different times. After
obtaining the results, the average errors of the control system for every ten sets of controller parameters are determined. In Table 7, $AE_{BBO-C}$, $AE_{BBO-C-\alpha}$, and $AE_{ABBO}$ are the average of ten various runs and $p-value$, which is determined by t-test of ANOVA, shows that diversity of the errors established by the three algorithms [51].

**Table 7.** Average error for BBO algorithms in radian.

<table>
<thead>
<tr>
<th>Joints</th>
<th>$AE_{BBO-C}$</th>
<th>$AE_{BBO-C-\alpha}$</th>
<th>$AE_{ABBO}$</th>
<th>$P-value$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip</td>
<td>0.0466</td>
<td>0.0438</td>
<td>0.0355</td>
<td>0.03297</td>
</tr>
<tr>
<td>Knee</td>
<td>0.0480</td>
<td>0.0382</td>
<td>0.0317</td>
<td>0.0384</td>
</tr>
</tbody>
</table>

The average errors for all the algorithms are less than 0.05 (rad), which is in the acceptable range [52], whereby the average error of trajectory error for ABBO is the lowest and BBO-C-\(\alpha\) is lower than BBO-C. This shows that applying BLX-\(\alpha\) as a migration operator improved the performance compared to the arithmetic migration operator; in addition, applying the adaptive variables instead of selecting constant values for BLX-\(\alpha\) migration operator improved the algorithm efficiency because it provides a dynamic exploration mechanism for ABBO algorithm to find the global optima and not being trapped in the local ones. The $P-value$ of the ANOVA test is less than 0.05, which expresses that the average error for ten times, running of algorithms is different from each other. Figure 13 represents the objective function that is in the first habitat as an elite, while the number of habitats is 40 and the three algorithms are run for 400 iterations.

For both hip and knee, ABBO has the fastest convergence, while BBO-C-\(\alpha\) with BLX-\(\alpha\) migration operator converges faster than BBO-C that uses arithmetic migration operator. Adaptive variables for BLX-\(\alpha\) lead the algorithm to reach the neighbor of the global optimum in a few iterations and explore the global optimum in limited searching space throughout the iterations.

For instance, the objective function reached 80 percent of its optimal value in 13, 39, and 63 iterations for ABBO, BBO-C, and BBO for the hip. Similarly, knee algorithms achieved 80 percent of the final objective function in 33, 57, and 130 iterations for ABBO, BBO-C, and BBO, respectively. These data show that ABBO converges faster because the parameters of the migration operator change proportional to the number of iteration, and starts with a larger range of searching space and narrows it to the global optima. Table 8 shows
the parameters of optimal PID controller by ABBO with a transfer function as a filter on a derivative section of the controller to reduce high-frequency noises created by the encoder.

Table 8. Parameters of optimal filtered PID.

<table>
<thead>
<tr>
<th></th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip</td>
<td>0.633</td>
<td>3.8251</td>
<td>6.9418</td>
<td>38.6768</td>
</tr>
<tr>
<td>Knee</td>
<td>0.3141</td>
<td>1.3625</td>
<td>2.7641</td>
<td>68.1906</td>
</tr>
</tbody>
</table>

To validate the performance of ABBO in an experimental platform, the closed-loop control system with the optimal controller by parameters in Table 8 is implemented in a real prototype of the LLE. The desired trajectory for hip and knee is a periodic trajectory that has a stand and swing phases of the gain training. Figure 14 shows the angular trajectory for hip and knee joints. The desired trajectories, determined for the rehabilitation gait training exercise, are inspired by the stand and swing phases of healthy human walking [11]. The desired trajectory equations for left hip, left knee, right hip, and right knee are expressed as follows:

$$y_{lh}(t) = max(-0.17\sin(ft + \pi), 0.35\sin(ft)),$$

$$y_{lk}(t) = min(0, 0.7\sin(ft + \pi)),$$

$$y_{rh}(t) = max(-0.17\sin(ft), 0.35\sin(ft + \pi)),$$

$$y_{rk}(t) = min(0, 0.7\sin(ft)),$$

where $t$ and $f$ are the elapsed time and frequency.

![Figure 14](image-url)  
**Figure 14.** Angular trajectories of (a) left hip, (b) left knee, (c) right hip, and (d) right knee.
Figure 15 represents the angular velocity for each joint.

Figure 16 shows the torque generated by actuators of left hip, left knee, right hip, and right knee.

**Figure 15.** Angular velocity of (a) left hip, (b) left knee, (c) right hip, and (d) right knee.

**Figure 16.** Torque of (a) left hip, (b) left knee, (c) right hip, and (d) right knee.
In Figure 16, when the trajectories are in the swing phase of gait training, more torque is applied to each joint. For all joints, the maximum generated torques did not exceed the maximum torque of the DC motor, which is $9.6 \text{Nm}$. Figure 17 illustrates the voltages applied to the hip and knee adjusted by the controller to lead each joint to pursue the desired trajectory.

![Graphs showing voltage applied to different joints.]

Figure 17. Voltage applied to (a) left hip, (b) left knee, (c) right hip, and (d) right knee.

In Figure 17, whenever the joints are in the stand phase, the controller issues the constant voltage through the actuators and while joints should follow the swing phase, the controller applies the required voltage for the actuators to produce the needed torque for each joint. Table 9 exhibits statistical analysis of error in radian for each joint, in which ME, AE, RSME, and IAE are represented.

<table>
<thead>
<tr>
<th>Joints</th>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ME</td>
<td>AE</td>
</tr>
<tr>
<td>Hip</td>
<td>0.058</td>
<td>0.020</td>
</tr>
<tr>
<td>Knee</td>
<td>0.131</td>
<td>0.024</td>
</tr>
</tbody>
</table>

AE is less than 0.05 (rad) that shows the acceptable range of error [52]. Table 10 shows the comparison of the performance of proposed ABB0 and PID controller with three other works.

For the current study IAE is higher than FLC-PID demonstrated in by 97% [29] and is less than adaptive-FLC-PID given by [30] by 10% for hip joint. However, the LLE model demonstrated in [29] and [30] are developed in Matlab/Simulink as the benchmark and they were not validated in the experimental prototype. In addition, the RMSE obtained in our work is 17% and 69% lower than PSO-PID represented in [28].
Table 10. Comparison of proposed ABBO and PID controller with others in the literature.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of tuning</td>
<td>ABBO</td>
<td>PSO</td>
<td>DFA</td>
<td>PSO</td>
</tr>
<tr>
<td>System model</td>
<td>4-DoF LLE</td>
<td>2-DoF LLE</td>
<td>4-DoF LLE</td>
<td>2-DoF LLE</td>
</tr>
<tr>
<td>Population size</td>
<td>40</td>
<td>20</td>
<td>N/A</td>
<td>20</td>
</tr>
<tr>
<td>No. of iteration</td>
<td>400</td>
<td>100</td>
<td>N/A</td>
<td>200</td>
</tr>
<tr>
<td>No. of design variables</td>
<td>4</td>
<td>3</td>
<td>N/A</td>
<td>3</td>
</tr>
<tr>
<td>IAE (rad)</td>
<td>0.263 (left hip)</td>
<td>N/A</td>
<td>0.0063 (left hip)</td>
<td>0.299 (hip)</td>
</tr>
<tr>
<td></td>
<td>0.286 (left knee)</td>
<td>N/A</td>
<td>0.01189 (left knee)</td>
<td>0.281 (hip)</td>
</tr>
<tr>
<td></td>
<td>0.250 (right hip)</td>
<td>N/A</td>
<td>0.0.0048 (right hip)</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>0.268 (right knee)</td>
<td>N/A</td>
<td>0.0108 (right knee)</td>
<td>N/A</td>
</tr>
<tr>
<td>RMSE (rad)</td>
<td>0.133 (left hip)</td>
<td>0.11 (hip)</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>0.147 (left knee)</td>
<td>0.045 (knee)</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

actuator used in [28] is BLCD Maxon which has the higher quality and power than the motor used in the present work.

8. Conclusion

In this paper, a PID controller was tuned by ABBO, in which the parameters of the migration operator changed proportionally to the number of iteration. The mathematical model of the LLE was determined by Lagrangian and Kirchhoff’s law and implemented in the control system. Tuning of the PID controller was established as an optimization problem based on minimizing the steady-state trajectory error and solved by ABBO. The results were compared with other conventional BBO to show their efficiency and appropriate performance. The tuned controller experimented with an actual LLE, and the results represented an acceptable range of trajectory error. The proposed optimization performed efficiently and the control system was validated for the actual LLE. However, a more robust adaptive controller can be used for avoiding the fluctuations in joints’ trajectory. This study can be extended to set optimal PID parameters in real time for the LLE application.

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References


