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Research Article

A game theoretic approach to design mating programs for livestock

Ebru ERGÜNEŞ BERKİN[1], Hasan ÖNDER[2,*], Özgür Hakan AYDOĞMUŞ[3]
[1]Islahiye Directorate of District Agriculture and Forestry, Gaziantep, Turkey
[2]Department of Animal Science, Faculty of Agriculture, Ondokuz Mayıs University, Samsun, Turkey
[3]Faculty of Political Science, Social Science University of Ankara, Ankara, Turkey

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Abstract: In this study, we proposed a new game theoretic method to design a mating program. The index and game theoretic methods were applied with calculated breeding values using pedigree on two different data sets whose economic traits consist of negative (milk yield and fat percentage) and positive (birth and weaning weight) genetic correlated data. For the negative genetic correlated data set, even if total expected benefits were equal for two methods, mating programs were changed, and the coefficient of variation obtained using the game theoretic method was smaller than that of the index method. This result showed that the expected breeding value will be more homogeneous in the next generation if the game theoretical approach is used. For the positive genetic correlated data set, the total expected benefit obtained from the index selection was a bit higher than the expected benefit obtained from the game theory. In terms of the coefficient of variation, selection of the index method provides 25% more homogeneous next generation flock structure than the game theoretic approach. When the results examined, it is clear that more studies should be done using game theoretical modeling, which is a new approach for animal mating design.

Key words: Game theory, animal mating design, breeding value, selection

1. Introduction

Game theory is a mathematical language for describing strategic interactions in which each player's choice affects the payoff of other players [1]. With a more simple sentence, game theory is a method examining to give the best response to expected strategies of their competitors for two or more players [2]. Individuals or groups that make decisions in the game are called players that can be considered as genes, people, companies, nation-states, etc. The players are assumed to be rational and take their knowledge and their expectations (beliefs) of the opponents' behavior into account [1].

In general, it is possible to classify games in two ways as chance and strategy games. Chance games are one player games, which played against nature. Strategy games can be played with two or more players. Strategy games can be classified according to result of the game; zero-sum games in which one's winnings are equal to the loss of the other or nonzero sum games that reveal balance situations that may be profitable on both sides. The games can be shown as flat-shaped games defined as normal or a tree in which the player's benefits are shown in a table where analogous considerations to the ones concerning strict dominance can be carried out for the elimination of weakly dominated strategies [3, 4]. Strategic game, normal game, and non-collaborative game expressions can be defined as similar expressions.

The expected benefit is the number of progenies or the number of copies of genes transferred to future generations [5]. Businesses engaged in livestock want to improve their livestock in order to maximize the benefits (offspring, meat, milk, wool, honey, etc.) they are expecting from the animals. Animal breeding aims to increase the genotypic value of the population in terms of the interested character in next generation [6]. The strategies that are expected to maximize benefit within a generation are built through both natural and breeding selection. High-yielding individuals are determined as breeding material by selection; hence, these animals have a chance to give more offspring to increase the benefit of next generation [7].

In the case of classical breeding, it is possible to calculate the individual index economic value for the females and males by using the selection index method, which is generally used in the case of more than one trait, and the mating programs can be formed by arranging these economic values descending. In this study, we propose a game theoretic approach for livestock production. At first, female and male animals within the same breed were described as players. According to the

* Correspondence: honder@omu.edu.tr

strategies they set out using the breeding values (BW) calculated from the pedigree records and phenotypic data using random regression test day model, the game was designed as a non-zero-sum and non-cooperative game. A pure Nash equilibrium (a Nash equilibrium is a set of strategies that players act out, with the property that no player benefits from changing their strategy, in other words, a Nash equilibrium is a profile of actions where each player's action is optimal given the actions of others) of the normal form game was found if possible. If not, a mixed strategy Nash equilibrium was found. These equilibria were used to match males and females to design a mating program. The purpose of this study is to investigate and apply a game theoretic method designing an animal mating program under the hypothesis that the game theory may be more useful for this aim than the first study in the literature. Then, we discuss the differences and similarities between the selection index and this new method by comparing the results obtained from both mating programs.

2. Materials and methods

2.1. Material

Two different data sets with negative and positive genetic correlations were used in the study. The first set of data is from a Jersey cattle breed with 5 head bulls and 50 head cows with BW's estimated for lactation milk yield (kg) and fat percentage (%), obtained from a commercial dairy farm for negative genetic correlation (–0.37). The second set of data is from Saanen goat breed with 5 head bucks and 50 head goats with BW's estimated for birth and sixth month weight, obtained from a commercial dairy goat farm for positive genetic correlation (0.47). In the realization of the analysis, MATLAB V.7.12.0.635 software was used with license number 161052. A used code was taken from Chatterjee [8].

2.2. Methods

Subjecting animals to selection is the only way that animal breeding can be accomplished. The animals are mated in the framework of a prepared mating design to obtain the next generation where the effect of the selection procedure can be seen. This selection can be performed by using the index method in cases where more than one trait is relevant. The index value can be calculated using the equation $I = W_A x A + W_B x B + ... + W_K x K$, where (W_A, W_B, W_K) are economic weights for traits (A, B and K indicated the phenotypes or BW). To design the mating, program all animals were arranged descending according to values and female animals determined to mate with male animals [9].

Alternatively, we now propose the game theoretic method to design an animal-mating matchup. In particular, we use bimatrix games whose set of players is denoted by N = {1, 2}, and their finite strategy sets are denoted by K and L. Then, one can describe the values of payoff functions by using a bimatrix as shown in Table 1 [10]. For numerical examples please check out (http://euler.fd.cvut.cz/predmety/game_theory/games_bim.pdf).

Here, $K = \{k_1, k_2, ..., k_m\}$ is the finite strategy set of player *1,* and $L = \{l_1, l_2, ..., l_n\}$ is the finite strategy set of player 2. Assume that *p1* and *p2* are payoff functions of player 1 and player 2, respectively. When the strategy pairs (*ki*, *lj*) were chosen, *aij*=*p1*(*ki*, *lj*) is the profit of player 1, and *b*ij=*p2*(*ki*,*lj*) is the profit of player 2 (http://euler.fd.cvut.cz/predmety/game_theory/games_bim.pdf).

2.2.1. Pure Strategy Nash Equilibrium

Let player set is $N = \{1, ..., n\}$ when n>1 in a non-cooperative game. *i*: any player, Si: strategy set of any player *i*, S_{-i}: strategy set of player other than player *i*, $S_1, ..., S_n$: set of all strategies and $P_1, ..., P_n$: payoff functions of players. A non-cooperative game can be defined as;

$G = (N, (S_i)_{i \in N}, (p_i)_{i \in N})$ or $G = \{S_1, ..., S_n, p_1, ..., p_n\}$.

For all $\boldsymbol{i \in N}$ for $S_i \neq 0$; p_i, $S = \coprod_{-i \in N} S_{-i}$;

$$p_i = \underbrace{S_1 x ... x S_n}_{S} \rightarrow R.$$

2.2.2. Best Response Function

$s \in S$, where s is any strategy profile. s_i: strategy set of any player *i*, s_{-i}: strategy set of player other than player *i*, s_i' : possible strategy of player *i* and $\phi_i(s_{-i})$: best strategy set of player *i*, hence the best response function can be defined as;

Table 1. Bimatrix illustration for normal form game with two players.

		Player 2			
		l_1	l_2	…	l_n
Player 1	k_1	(a_{11}, b_{11})	(a_{12}, b_{12})		(a_{1n}, b_{1n})
	k_2	(a_{21}, b_{21})	(a_{22}, b_{22})		(a_{2n}, b_{2n})
	⋮				
	k_m	(a_{m1}, b_{m1})	(a_{m2}, b_{m2})		(a_{mn}, b_{mn})

$$\phi_i(s_{-i}) \equiv \arg\max_{s_i \in S_i} p_i(s_i, s_{-i})$$
$$\equiv \{s_i \in S_i : p_i(s_i', s_{-i}) \geq p_i(s_i, s_{-i}) \forall \in S_i\}$$

For a game with two players, best responses of both players to other can be defined as;

$s_1^* = \phi(s_2^*)$: s_1^* is the strategy profile of player 1

$s_2^* = \phi(s_1^*)$: s_2^* is the strategy profile of player 2

here it should be,

$$\frac{\partial p_i(s_1, \ldots, s_n)}{\partial s_i} = 0, \text{ i=1,...,n and } \frac{\partial p_i}{\partial s_i^2} \leq 0.$$

The equilibrium obtained by solving the *n* equations is Nash equilibrium. The fact that the s^* strategy profile is the Pure Strategy Nash equilibrium is because the strategy of a player is the best response to the strategies of other players. $s^* = (s_1^*, s_2^*)$ only if $(s^* \in \varnothing(s^*))$.As a result, the pure strategy Nash equilibrium is as described in the following equation;

$$p_i(s_i^*, s_{-i}^*) \geq p_i(s_i, s_{-i}^*), \ \forall_i \in N, \ \forall s_i \in S$$

2.2.3. Iterated elimination of strictly dominated strategies

Strictly dominated strategy is a strategy that a rational player will not play [11, 12]. Since each player knows that the other player will not play the strictly dominant strategy, these strategies are deleted from the game. This process is named as "Iterated Elimination of Strictly Dominated Strategies" and defined with an algorithm that has an iteratively shrinking strategy set S_i^k (k=0,1,2...) for each player $i \in N$. In every step of the deletion of strictly dominated strategies, a new game is obtained, and this process ends in the fourth step:

Step 1

$S_i^0 = S_i$ for k=0

Step 2

$$S_i^1 = \left\{ s_i \in S_i^0 \setminus \nexists s_i' \in S_i^0 p_i(s_i', s_{-i}) > p_i(s_i, s_{-i}) \forall s_{-i} \in S_{-i}^0 \right\}$$

Step 3 k+1

$$S_i^{k+1} \left\{ s_i \in S_i^k \setminus \nexists s_i' \in S_i^k p_i(s_i', s_{-i}) > p_i(s_i, s_{-i}) \forall s_{-i} \in S_{-i}^k \right\}$$

Step 4

$$S_i^{\infty} = \bigcap_{k=1}^{\infty} S_i^k$$

2.2.4. Mixed Strategy Nash Equilibrium

In some games, there is no pure strategy Nash equilibrium. To find the Nash equilibrium, a mixed strategy must be used, and this game is played by randomizing the strategies.

When $\sum_i$ is mixed strategy space of any player *i*, S_i is a set of probability distributions over $\sum_i$. Thus, if $\sigma_i \in \sum_i$, then $\sigma_i(s_i)$ is the probability that player *i* chosen action s_i.

Probability distribution is $s_i : S_i \to [0,1]$ over finite not null set of S_i. So,

$$\sum_{s_i \in S_i} \sigma_i(s_i) = 1.$$

Let σ is any mixed strategy profile, σ_{-i} is mixed strategy of players other than i, σ_i' is possible strategy of player i, σ_{-i}' is possible strategy of players other than i, σ_i^* is mixed strategy profile of player i, σ_{-i}^* is mixed strategy profile of players other than i, σ^* is mixed strategy profile and $\phi_i(\sigma_{-i})$ is the best mixed strategy set of player i; to explain mixed strategy space, the payoff on mixed strategy can be defined as follows:

$$p_i(\sigma_i, \sigma_{-i}) = \sum_{s_i \in S_i} p_i(s_i, \sigma_{-i}) \sigma_i(s_i)$$
$$= \sum_{s \in S} p_i(s_i, s_{-i}) \left(\prod_{j=1}^{n} \sigma_j s_j \right)$$

This payoff function can be interpreted that the expected payoff should be von-Neumann Morgenstern (VNM) for chosen strategies of players.

$$\phi_i(\sigma_{-i}) \equiv \arg\max_{\sigma_i \in \sum_i} p_i(\sigma_i, \sigma_{-i})$$
$$\equiv \left\{ \sigma_i \in \sum_i : p_i(\sigma_i', \sigma_{-i}) \geq p_i(\sigma_i, \sigma_{-i}) \forall \sigma_i \in \sum_i \right\}$$

For example, for a two-player game, the Nash equilibrium of a σ^* mixed strategy is due to the fact that a player's mixed strategy is the best response to the mixed strategies of other players.

$$\sigma^* = (\sigma_1^*, \sigma_2^*)$$

Mixed strategy Nash equilibrium can be defined as (https://warwick.ac.uk/fac/soc/economics/staff/dsgroi/ec202/w06_dominant_strategies_and_iesds.pdf; http://www.sam.sdu.dk/~psu/teaching/phd/draft.pdf);

$$p_i(\sigma_i^*, \sigma_{-i}^*) \geq p_i(\sigma_i, \sigma_{-i}^*), \quad \forall_i \in N, \quad \forall \sigma_i \in \sum_i$$

Here, we suppose that the row players are males, and the column players are females. The payoff matrix is constructed by adding the breeding values of individual male and female animals. If we want to optimize the milk yield and fat percentage of *i*th male and *j*th female animals, for instance, then *aij* (*bij*) is the addition of *i*th male and *j*th female animals milk (fat) characteristic values.

Then, one finds the Nash equilibrium of the game to match each male with a desired number of female animals. When *j*th female is matched with a male the *j*th row of the bimatrix is deleted. When ith male is matched with a desired number of females, the *i*th column is deleted from the bimatrix. This process is continued until all animals are matched.

To compare the expected benefits (EB) and coefficient of variation (CV) between methods Mann–Whitney U test was used [13]. The variability of CV was calculated among sires.

3. Results

The selection index method applied to more than one character in classical breeding is calculated as a linear combination of individual breeding values and maximization is aimed at selection [14, 15]. The benefits obtained in the negative genetic correlation scenarios were shown in Table 2 for the selection index and in Table 3 for the game theory methods. The benefits obtained in the positive genetic correlation scenarios were shown in Table 4 for the selection index and in Table 5 for the game theory methods.

When the mating design generated by the selection index and game theory is examined in terms of the traits with negative genetic correlations between them, it is seen that the animals selected for breeding are the same animals in both methods. In other words, the mating method didn't affect the selection of the animals for breeding. But different mating couples of animals were observed for methods. Results showed that the total

Table 2. The expected benefits of the mating program generated by the selection index in terms of traits have negative genetic correlations for Jersey cattle.

Bull Cow	101	106	110	111	122
1	–104.2	137.4	–0.8	179.3	254.9
2	–106.2	135.3	–1.1	175.5	249.5
3	–121.6	120.8	–1.9	169.5	247.8
4	–128.8	119.2	–3.9	168.8	230.1
5	–132.5	116.7	–9.8	167.3	218.8
6	–133.1	113.5	–11.4	164.56	215.7
7	–137.4	111.4	–24.9	162.1	200.1
∑EBi	–863.8	854.3	–53.9	1187.2	1616.9
CV	10.8	8.4	113.2	3.6	8.9
∑EB	2740.6				
CV	166.3				

The numbers at the first row and column shows the animal ID.

Table 3. The expected benefits of the mating program generated by the game theory in terms of traits have negative genetic correlations for Jersey cattle.

Bull Cow	101	106	110	111	122
1	–19.3	106.6	114.1	155.5	109.8
2	–23.1	106.2	108.7	153.4	107.8
3	–29.1	105.5	106.9	138.9	92.4
4	–29.9	103.4	89.2	137.3	85.2
5	–31.4	97.6	77.9	134.8	81.6
6	–34.1	95.9	74.8	131.7	80.9
7	–36.5	82.4	59.3	129.5	76.6
∑EBi	–203.4	697.7	630.9	981.1	634.3
CV	20.7	8.8	22.8	7.3	14.7
∑EB	2740.6				
CV	75.1				

The numbers at the first row and column shows the animal ID.

Table 4. The expected benefits of the mating program generated by the selection index in terms of traits have positive genetic correlations for Saanen goat.

Buck Goat	101	102	110	115	119
1	–0.7	0.2	–0.5	1.6	2.7
2	–0.7	0.1	–0.5	1.5	2.6
3	–0.7	0.1	–0.5	1.5	2.5
4	–0.8	0.0	–0.6	1.5	2.5
5	–0.9	0.0	–0.6	1.5	2.3
6	–0.9	0.0	–0.7	1.5	2.2
7	–0.9	–0.2	–0.7	1.5	2.2
∑EBi	–5.7	0.3	–4.0	10.4	16.9
CV	12.0	287.6	14.2	2.3	8.1
∑EB	17.9				
CV	247.2				

The numbers at the first row and column shows the animal ID.

Table 5. The expected benefits of the mating program generated by the game theory in terms of traits have positive genetic correlations for Saanen goat.

Buck Goat	101	102	110	115	119
1	–0.0	–1.1	–1.5	2.0	2.7
2	–0.7	0.3	–1.4	1.7	2.5
3	–0.3	0.3	–0.2	1.3	2.2
4	0.0	–0.5	–0.5	0.9	2.6
5	–0.7	–1.0	–1.0	1.0	2.0
6	–0.5	0.3	–1.1	0.8	2.3
7	–0.3	0.2	–0.9	1.5	1.7
∑EBi	–2.5	–1.6	–6.5	9.2	15.9
CV	81.1	271.1	49.3	32.5	15.3
∑EB	14.4				
CV	310.0				

The numbers at the first row and column shows the animal ID.

Table 6. Comparison of selection index and game theoretic approaches.

	Negative genetic correlation		Positive genetic correlation	
	EB	CV	EB	CV
Selection index	548.1 ± 447.24	28.9 ± 21.08	3.6 ± 4.35	64.8 ± 55.71
Game theory	548.1 ± 198.60	14.9 ± 3.09	2.9 ± 4.17	89.9 ± 46.59
P for Mann Whitney U	0.60	0.60	0.75	0.12

expected benefits were equal (2740.63) for both methods. This may be caused from the fact that selected animals for breeding were same for both methods. When variation coefficient (CV) was examined, it can be seen that the value obtained from the mating design realized by the game theoretic approach was much lower than the value obtained from the mating design according to the selection index. This result can be regarded an indicator of a more homogeneous expected benefit that can be obtained at the new generation from the mating program realized by the game theoretic approach than selection index. Comparison of the selection index and game theoretic approaches were given in Table 6.

4. Discussion

Although the index method is still popular for its various advantages nowadays, it is difficult to calculate the values used in the calculation of the index equation, which contains high sampling error, the contribution of each genotype has different effect on population genotypes and the maximization of the individuals obstructed the calculation of economic contribution to the population. It also brings disadvantages that one of the most prominent problems of the selection index method is that the traits or yields that enter the index while individuals selected can change out of control from positive to negative [9, 15] This problem has been overcome since the expected benefit of the population is optimized in the developed game theoretic approach.

In the comparison by expected benefit and coefficient of variation (CV) for negative genetic correlated data, for the bull with ear number of 110, the expected benefit was higher (–53.94<630) in the game theoretic approach than the index method, and CV was lower (166.28>75.11) in the game theoretic approach than the index method. In the index method, the expected benefit of the bull 122 was decreased nearly 61% and CV was increased nearly two times when comparing with the game theory. This result shows that the game theory method is more likely to provide a number of advantages such as a more homogenous mating design and, thus, the increase of the desired genotypes in the population in the sense of animal breeding and the simplicity of maintenance and feeding conditions and the ease of herd management in terms of raising animals.

For positive genetic correlated data, selected animals for breeding were not same for two methods. Only 82.86% of animals were same for mating selection. It was found that the expected benefit obtained by the index method was a bit higher (17.913>14.438) than the game theoretic approach. When the CV was examined, it was found that the index method was 25% homogeneous than the game theoretic approach. For both sets of data that contain both negative and positive genetic correlations, the common feature of the two methods is that the best optimizing bull / buck is over after the other bull / buck. This leads to the conclusion that the data may be related to cardinal values (numerical quantities).

According to the results obtained, it is understood that the game theoretic approach produces homogenous next generation expectancy especially when it is aimed to perform selection and mating design in terms of features having negative genetic correlation between them. The homogeneity of the expected utility of the next generation in animal breeding is gaining importance, which is why the variance of the response given to the environmental conditions that will arise will be reduced, and, therefore, the environmental conditions can be controlled more easily [16]. The homogeneity of the trait to be breed also increases the success of the statistical methods used in animal breeding [17]. The optimization of both sexes is more important in animal breeding, especially for fattening characters, even if used methods are based on the selection of male individuals and their maximization without any expected benefit loss.

While there is individual benefit in the selection index, population utility is the forefront in the game theoretic approach. Use of the game theory may be more beneficial on the populations that desired breeding aims have been nearly reached. It is desired to increase the homogeneity in the obtained progeny population with using the game theoretic approach. In this way, it will be possible to manage the environmental conditions much more easily in practice, and the operating costs can be reduced. In this respect, it is important that the offspring population can be obtained homogeneously [6]. Taking all the analysis results into consideration, it is quite clear that there is a need for further study on the game theory which was a new approach to animal mating designs in order to validate the efficiency of the game theoretic approach on experimental breeding studies to show the methods superiority for optimization and maximization.

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