

## A new method for optimal expansion planning in electrical energy distribution networks with distributed generation resources considering uncertainties

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**Abstract:** The present study aims to introduce a robust model for distribution network expansion planning considering system uncertainties. The proposed method determines optimal size and placement of distributed generation resources, as well as installation and reinforcement of feeders and substations. This model is designed to minimize cost and to determine the best time for the installation of equipment in the expansion planning. In the proposed expansion planning, the fuzzy logic theory is employed to model uncertainties of loads and energy price. Also, since the proposed model is a nonlinear and nonconvex optimization problem, a tri-stage algorithm is developed to solve it. Simulation results revealed that the proposed model would be capable to improving the performance of the expansion planning in distribution networks.

**Key words:** Uncertainty, reliability, multistage expansion planning, optimal power flow, distributed generation resources

### 1. Introduction

Today, distribution network expansion planning (DNEP) is one of the interesting topics in academia. If proper strategies are selected, distribution companies (DISCOs) can guarantee power supply in the future and provide their subscribers with cheap and high-quality electricity; therefore, they must make suitable DNEP. This expansion can be met by combining novel methods with traditional power distribution strategies [1]. Distributed generation resources (DGRs) are recognized as a novel and powerful strategy to enhance network performance and energy cost [2]. Most previous studies have included traditional expansion options [3–7], while this paper investigates DNEP and DGRs simultaneously.

Researchers have introduced some optimal and suboptimal algorithms to accomplish DGRs in the distribution networks. For this purpose, continuous and discrete parameters should be considered [8]. However, due to extensive network coverage, there is an extremely large number of these parameters [9]. In addition, due to nonlinearity behavior of distribution networks and the presence of more than one local optimum point, the optimization problem is prominent. It is worth noting that nonlinear and nonconvex load flow complicates calculations [10–13]. Also, a serious challenge is raised by incorporating uncertainty into the DNEP problem. Given the two former features, the DNEP studies can be categorized into four groups. First, methods with AC or DC linear load flow, wherein linear approximation lowers the efficiency of the algorithms [14]. Second, methods that take nonlinear load flow into account, wherein the computational burden of the problem is escalated [15]. Third, methods that disregard uncertainty due to its effect on further complexity of the optimization process

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[16]. These methods are not sufficiently accurate. In other words, if an uncertain parameter deviates from its forecasted value, the obtained optimal solutions may be nonoptimal. Thus, uncertainty modeling increases the robustness of optimal solution. Fourth, stochastic methods that take uncertainty into account, wherein input data is expanded and the computational complexity is elevated [17].

The proposed method in this paper, is a promising tool for the achievement of optimal results, as compared to the first three former categories, and reduces the computational burden compared to the fourth category. The fuzzy approach is used to model system uncertainties, including power demand and energy price. This method ensures the required modeling accuracy while the computational burden does not increase significantly. In other words, compared to probabilistic methods, the proposed approach reduces the number of iterations to solve the problem. On the other hand, the fuzzy method is not restricted by information about the network in the previous years.

For the assessment of the system reliability is considered reserve feeders and the possibility of islanding after a fault.

This study aims to develop a multiobjective algorithm for the simultaneous expansion of the distribution network and DGRs. The main contributions of this paper can be summarized as follows:

1. Proposing a multistage robust model for multiyear DNEP by incorporating the uncertainty sources of the energy price and the load.
2. Using a dynamic method for the DNEP problem, wherein the timing of the expansion option installation is addressed.
3. The proposed model benefits from a techno-economic procedure to meet two goals, namely "minimization of investment and operation costs" and "enhancement of the reliability".

The rest of the paper is organized as follows. Section 2 describes modeling of the objective function and its constraints. In Section 3, the proposed method for solving the DNEP problem is expressed in detail. The simulation results and conclusion are described in Sections 4 and 5, respectively.

## 2. Modeling of the DNEP problem

### 2.1. Objective function

In the proposed approach, the role of the DNEP objective function is to minimize the total cost represented in Equation (1) [18].

$$OF = TIC_y + TOC_y + TMC_y + TCOC_y \quad (1)$$

Usually, DISCOs suffer a limited budget, which needs to be considered in the DNEP. Equation (2) shows the investment cost to install the selected expansion options. Also, Equation (3) indicates that investment cost has three terms, including installation of substations, feeders, and DGRs.

$$TIC_y = \sum_{p=1}^{N_p} \left( \frac{(1 + Ir)^\delta}{(1 + Ir)^\delta - 1} \times (1 + Ir)^{-p} \right) \times C_{y,p}^{inv} \quad (2)$$

$$\begin{aligned}
 C_{y,p}^{inv} = & \sum_{s=1}^{N_s} \beta_s^{rein} \cdot C_{s,p}^{rein} + \sum_{f=1}^{N_f} \beta_f^{rein} \cdot [C_{f,p}^{rein} \cdot FL_f] \cdot (USF(p - Y_i^{rein} + 1) - USF(p - Y_i^{rein})) + \\
 & \sum_{f=1}^{N_f} \beta_f^{ins} \cdot [C_{f,p}^{ins} \cdot FL_f] + \sum_{d=1}^{N_d} \beta_d^{ins} \cdot [C_{d,p}^{ins} \cdot (P_d^{cap} + P_d^{res})] \cdot (USF(p - Y_i^{ins} + 1) - USF(p - Y_i^{ins}))
 \end{aligned} \quad (3)$$

If DGR is installed at bus  $i$ ,  $\beta_d^{ins} = 1$ , and otherwise  $\beta_d^{ins} = 0$ .

If the feeder is reinforced at bus  $i$ ,  $\beta_f^{rein} = 1$ , and otherwise  $\beta_f^{rein} = 0$ .

Operation cost is obtained by Equation (4), wherein the present value of the operation cost in each year of planning horizon has been modeled using interest rate factor. It is worth noting that there is a difference between the values of investment cost interest rate and operation cost interest rate. Equation (5) includes the costs of energy losses and purchasing energy from the upstream network.

$$TOC_y = \sum_{p=1}^{N_p} \left( \left( (1 + Ir)^{-p} + \frac{(1 + Ir)^{-Np}}{Ir} \right) \cdot \sum_{l=1}^{N_l} \left( 8760 \times EMC_l \times \frac{LLD_l}{24} \right) \times C_{y,p}^{oper} \right) \quad (4)$$

$$C_{y,p}^{oper} = \sum_{s=1}^{N_s} R_s \times [I_{s,l,p}^{oper}]^2 + \sum_{f=1}^{N_f} R_f \times [I_{f,l,p}^{oper}]^2 + \sum_{d=1}^{N_d} \sigma_{d,l,p}^{oper} \times P_{d,l,p}^{oper} + \sum_{s=1}^{N_s} \gamma_{s,l,p}^{oper} \times P_{s,l,p}^{oper} \quad (5)$$

Equation (6) includes the cost of repair and maintenance of the network equipment. Equation (7) indicates that repair and maintenance costs are induced by substations and feeders.

$$TMC_y = \sum_{p=1}^{N_p} \left( \left( (1 + Ir)^{-p} + \frac{(1 + Ir)^{-Np}}{Ir} \right) \times C_{y,p}^{maint} \right) \quad (6)$$

$$C_{y,p}^{maint} = \sum_{s=1}^{N_s} C_{s,p}^{maint} + \sum_{f=1}^{N_f} C_{f,p}^{maint} \quad (7)$$

Equation (8) calculates the expected customer outage cost over the planning horizon. Also, the customer outage costs in period  $p$  are obtained via Equation (9).

$$TCOC_y = \sum_{p=1}^{N_p} \left( \left( (1 + Ir)^{-p} + \frac{(1 + Ir)^{-Np}}{Ir} \right) \times C_{y,p}^{cu-out} \right) \quad (8)$$

$$C_{y,p}^{cu-out} = \sum_{l=1}^{N_l} 8760 \cdot \frac{LLD_l}{24} \cdot \sum_{i=1}^{N_i} \sum_{e=1}^E P_{i,m,l,p}^e \times f_{i,m,l,p}(r_{i,e}) \times \rho_{i,m}^e \quad (9)$$

In the present paper, TCOC is assumed as a criterion to assess network reliability. For calculation of this component, the possibility of islanding after a fault is incorporated into a failure event. Moreover, the load flow is performed on this island, followed by checking all of the relevant constraints. Let a fault occur in feeder  $l$ , the faulted section is then isolated from the rest of the network. It is worth noting that the disconnected sections have connected loads, whereby two cases can be emerged:

Case 1: One DGR exists in the isolated part. This section is assumed as an island to meet the isolated loads until the faulted section is repaired, provided that DGR has enough capacity and can satisfy the relevant constraints. Otherwise, the loads of island are shed until the output power of DGR becomes less than its capacity.

Case 2: There is a reserve feeder between the isolated part and the rest of the network, which can connect the island to neighboring feeders and supply the island's loads while satisfying relevant constraints. Otherwise, all loads of the isolated part are not supplied until repair of the faulted section.

## 2.2. Constraints on the problem

In this paper, three categories of constraints are considered in the DNEP problem, which are defined mathematically in follows [19].

### 2.2.1. Operation constraints

Generally, operation constraints are dependent on the standards of equipment, which are described in Equations (10)–(14). Equations (10) and (11) express constraints on the current of feeders and substations, respectively. Voltage constraint is defined by Equation (12), and constraints on the capacity of DRGs are described by Equations (13) and (14).

$$0 \leq I_{f,p}^{oper} \leq I_{f,p}^{max} \quad (10)$$

$$0 \leq I_{s,p}^{oper} \leq I_{s,p}^{max} \quad (11)$$

$$V_{i,l,p}^{min} \leq V_{i,l,p} \leq V_{i,l,p}^{max} \quad (12)$$

$$(\beta_d^{ins} \cdot USF (p - Y_i^{ins} + 1)) \cdot P_d^{min} \leq P_{d,p} \leq (\beta_d^{ins} \cdot USF (p - Y_i^{ins} + 1)) \cdot P_d^{max} \quad (13)$$

$$(\beta_d^{ins} \cdot USF (p - Y_i^{ins} + 1)) \cdot Q_d^{min} \leq Q_{d,p} \leq (\beta_d^{ins} \cdot USF (p - Y_i^{ins} + 1)) \cdot Q_d^{max} \quad (14)$$

### 2.2.2. Constraints on Kirchhoff's circuit laws

Kirchhoff's circuit laws play essential roles in the electrical engineering; thus, they need to be considered in the proposed method. All of the algorithms should always satisfy these laws in all situations. In Equations (15) and (16), these laws have been regarded as constraints.

$$IM^\gamma \times I_{l,p} + IN_{l,p} = ED_{l,p} - EDC_{l,p} \quad (15)$$

$$IM_{row,f}^\gamma \times V_{l,p} + Z_f \times I_{f,l,p} = 0 \quad (16)$$

### 2.2.3. Economic constraints

Economic constraints are imposed on the budget, restricting expansion of the network [20]. Equation (17) is related to DISCO in period  $p$  and Equation (18) indicates the constraint on the budget in the entire network expansion planning horizon.

$$C_{y,p}^{inv} \leq TB_p \quad (17)$$

$$\sum_{p=1}^{Np} C_{y,p}^{inv} \leq TB \quad (18)$$

### 2.3. Distribution network load flow

Load flow is an important tool for investigating the distribution network, where DNEP problem optimization requires successive load flows [21]. In this regard, a method must be developed with appropriate convergence speed. In this paper, a load flow method is proposed based on the backward/forward sweep. In this method, voltage of other buses and current of feeders are obtained (forward sweep) by starting from the last bus of network and allocating arbitrary value (usually the nominal voltage value) to the substation. Then, the new voltage is determined for other buses (backward sweep) by allocating the nominal voltage to the first bus (substation) and using calculated current for the feeders in the forward sweep step. The process continues until the desired bus voltage error is met.

### 2.4. Uncertainties

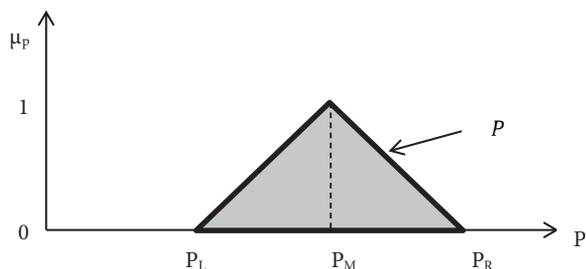
Due to the network vastness, installation of the measuring equipment at all load points is impossible. Also, estimation methods are usually used to evaluate loads in the network, which depend on several unpredictable factors such as behavior of the customers [22]. Since load uncertainty can improve network model, the probability theory is a conventional method to estimate uncertainty. However, the probability theory-based models are restricted by information about the network in previous years. Besides, the fuzzy logic method can be used to model uncertainty in the network loads [23]. In this model, power demand in each load point is described by triangle fuzzy number, as shown in Figure 1. Each triangle fuzzy number has three parameters ( $P_L, P_M, P_R$ ) that show the expected load ( $P_M$ ), minimum load ( $P_L$ ), and maximum load ( $P_R$ ). Let  $P_0$  be the estimated power at load point and  $\varepsilon$  be the maximum error. Equations (19)–(21) indicate the fuzzy parameters corresponding to  $P_0$ .

$$P_L = P_0 \times (1 - \varepsilon) \quad (19)$$

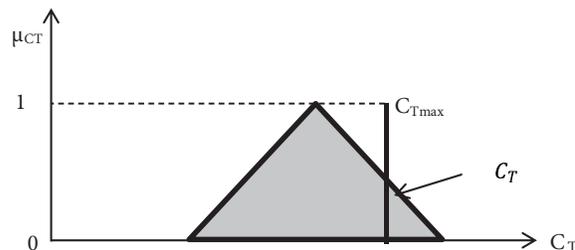
$$P_M = P_0 \quad (20)$$

$$P_R = P_0 \times (1 + \varepsilon) \quad (21)$$

DISCOs exchange electricity energy with others or costumers. Energy price changes based on several factors, which introduces uncertainty in the electricity market [24]. Also, energy price is restricted by government policies. Figure 2 shows a triangular fuzzy number for modeling energy price uncertainty.



**Figure 1.** Graphical representation of the load as a triangular fuzzy number.



**Figure 2.** Graphical representation of the energy price as a triangular fuzzy number.

### 3. Proposed solution method

In the optimization problem with multiple metrics, each metric has a certain optimization point that should be reached simultaneously. Therefore, rather than just one solution, a set of solutions is available. The final optimal solution in this set depends on the significance of these metrics in practical terms. In other words, in the multiple optimization algorithms with  $m$  objective functions, solution  $X$  dominates solution  $Y$ , if  $X$  is better than  $Y$  at least in one objective function and not worse in other objective functions as well. In this paper, GA-OPF was introduced as an appropriate multiobjective optimization method coupled with fuzzy theory.

#### 3.1. Optimization based on GA-OPF

In the genetic algorithm (GA), the total population of solutions is applied instead of a single solution. In other words, GA simultaneously processes more than one solution and updates populations in each epoch to obtain a global solution [25]. GA codes use discrete search space, even though the function may be continuous, which creates some difficulties such as the consumption of more computational power and lack of convergence into near-optimal solutions. Also, GA cannot reach any arbitrary precision. A hybrid coded GA was applied in this research to overcome these difficulties, with the advantages of both binary and real codes [26]. In this algorithm, decision variables can be coded in finite length strings, and portions of parent strings can be exchanged more easily to form new string. As the advantages of real codes, real parameters can be used intactly, and crossover and mutation operators can be directly applied to the parameter values.

#### 3.2. Optimization in presence of the uncertainties

The robustness of the optimal solution is dramatically affected by uncertainty modeling. Various approaches have been developed in the literature to immunize the distribution expansion plan against the worst case realizations of the uncertain parameters. The solution achieved by the proposed approach has robustness against realization of the uncertain parameters if the uncertainty evaluation strategy fits the solution algorithm. In this paper, the fuzzy method is implemented for characterization of the uncertainties and specifying a flexible and robust expansion plan.  $f_{T_k}$  and  $f_{E_k}$  stand for fuzzy solutions under load and energy price uncertainties, respectively. Also,  $f_{Tmax}$  and  $f_{Emax}$  denote the maximum fuzzy solutions for uncertainty states. In the fuzzy model, for set of  $(f_{Tmin}, f_{Emin})$  the optimal solution is  $(1, 1)$  and the worst solution for set of  $(f_{Tmax}, f_{Emax})$  is  $(0, 0)$ .  $\mu_k$  is the normalized value of nonlinear components of the objective function,

shown in Equation (22).

$$\mu_k = \left( \frac{f_{Tmax} - f_{Tk}}{f_{Tmax} - f_{Tmin}}, \frac{f_{Emax} - f_{Ek}}{f_{Emax} - f_{Emin}} \right) \quad (22)$$

The above equation must satisfy maximum of the defined uncertainty set. k indicates the number of effective answers. Figure 3 shows the uncertainty assessment based on the fuzzy theory with GA-OPF.

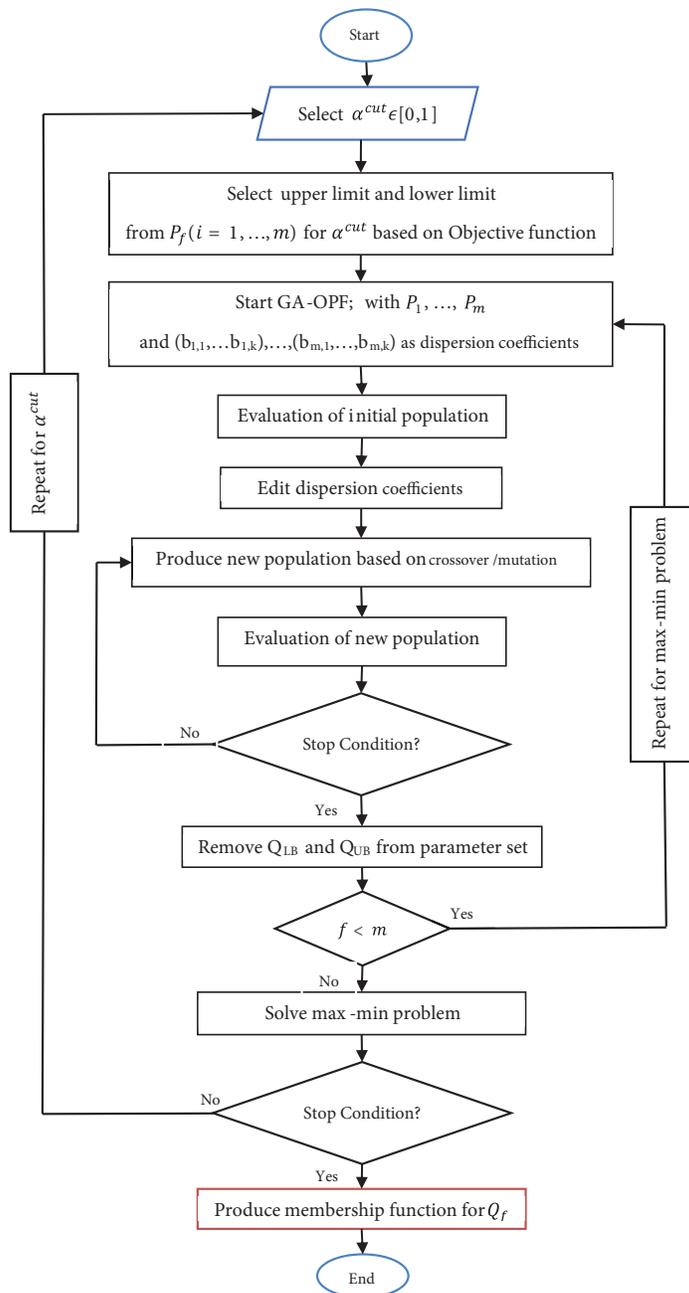


Figure 3. Process of modeling the uncertainties based on the fuzzy theory incorporation with GA-OPF algorithm.

### 3.3. Proposed chromosome structure

All information about the distribution network structure needs encoding in the chromosome genes to optimize the proposed DNEP problem. The proposed chromosome consists of three parts. The first part includes decision information about substation sizing and siting. This part has  $n$  genes, where  $n$  stands for the number of candidates and available substations. In each gene, integer number indicates the size of the respective substation and a zero value shows that the corresponding substation has not been installed.

The second part of the chromosome represents the network topology and feeder routes. This part includes three strings; each has  $nf$  genes, where  $nf$  represents the number of candidates and available feeders. The structure of feeders is indicated in the first string based on integer permutation encoding. The value of this string shows  $nf$  different feeders ranging between one and  $nf$ . Also, the structure of the network is shown by the sequence of these integers as follows. At the first step, no feeder is assumed to be installed. Then, by starting with the first integer in the string, the corresponding feeder is installed. This procedure is repeated for all integers in the string. For each integer in the chromosome, the feeder installation is restricted if it does not abide by the radial constraint. The second string in this part indicates the conductor type, wherein integer numbers in the genes apply conductor size number to the corresponding feeder in the first section. In this part, the last string shows reserve feeder sections, where each gene has two values, 1 or 0. Value 1 shows that the abandoned feeder section has been installed as a reserve section.

The third part of the chromosome accounts the capacity and location of DGRs. Also, this part includes  $ng$  genes, where  $ng$  shows the number of suitable locations for DGR installation. In each gene, an integer number shows the size of the selected DGR at the respective location.

### 3.4. Proposed method for the DNEP problem

The DNEP is a complex optimization problem involving discrete and continuous variables, which can be more complicated by incorporating the uncertain nature of some vital variables, such as the load and the energy price. In order to solve this problem, it is necessary to implement advanced methods to reduce the computational burden and maintain the required model accuracy at the same time [27]. For this purpose, a tri-stage decomposition approach is proposed, which are described in Equations (23)–(26).

$$\min_{R \in \mu^{IS}} (\alpha' \cdot R + \max_{U \in \mu^{US}} \min_{\Gamma \in \sigma^{AC-OPF}(R,U)} \beta' \cdot P) \quad (23)$$

where:

$$S_I) \quad \mu^{IS} = \{R \in \{0, 1\}^{N_R} \mid AR \geq B\} \quad (24)$$

$$S_{II}) \quad \mu^{US} = \{U \in R^{N_U} \mid \hat{u} - \hat{U} \leq U \leq \hat{u} + \hat{U}\} \quad (25)$$

$$S_{III}) \quad \sigma^{AC-OPF} = \{P \in R^{N_P} \mid I(R, P, U) \geq 0\} \quad (26)$$

The proposed DNEP model can be defined as the min-max-min optimization problem. In this problem, efforts are made for the minimization of the objective function subjected to the worst case realization of the uncertain parameters. The proposed algorithm can be summarized as follows:

1. In the first-stage, a MILP problem is solved, identifying the placement, timing, and optimal size for installation or reinforcement of DGRs, substations, and feeders.

2. In the second-stage problem, a multiyear max-min linearized AC optimal power flow (LAC-OPF) is solved to reach the worst-case realizations of energy price and loads. The algorithm returns to step 1 provided that there is not a satisfying convergence criterion between the first- and second-stage problems. Otherwise, the optimal investment plan and the worst case realizations of uncertain parameters obtained by solving the first- and second-stage problems, respectively, are submitted to the third-stage problem.
3. In the third-stage problem, nonlinear AC power flow equations are incorporated; therefore, involving other components of the objective function. Then, a multiyear nonlinear AC optimal power flow problem is solved. If there is not a satisfying convergence criterion between the first- and third-stage problems, the algorithm returns to step 1.

Figure 4 depicts the flowchart of the tri-stage proposed method. The proposed model comprises the binary variables  $(\beta_d^{ins}, \beta_f^{ins}, \beta_s^{rein}, \beta_f^{rein}, \dots)$ , integer variables  $(Y_i^{rein}, Y_i^{ins}, \dots)$ , and also real variables  $(P_{d,l,p}^{oper}, \dots)$ , introduced in Section 2.

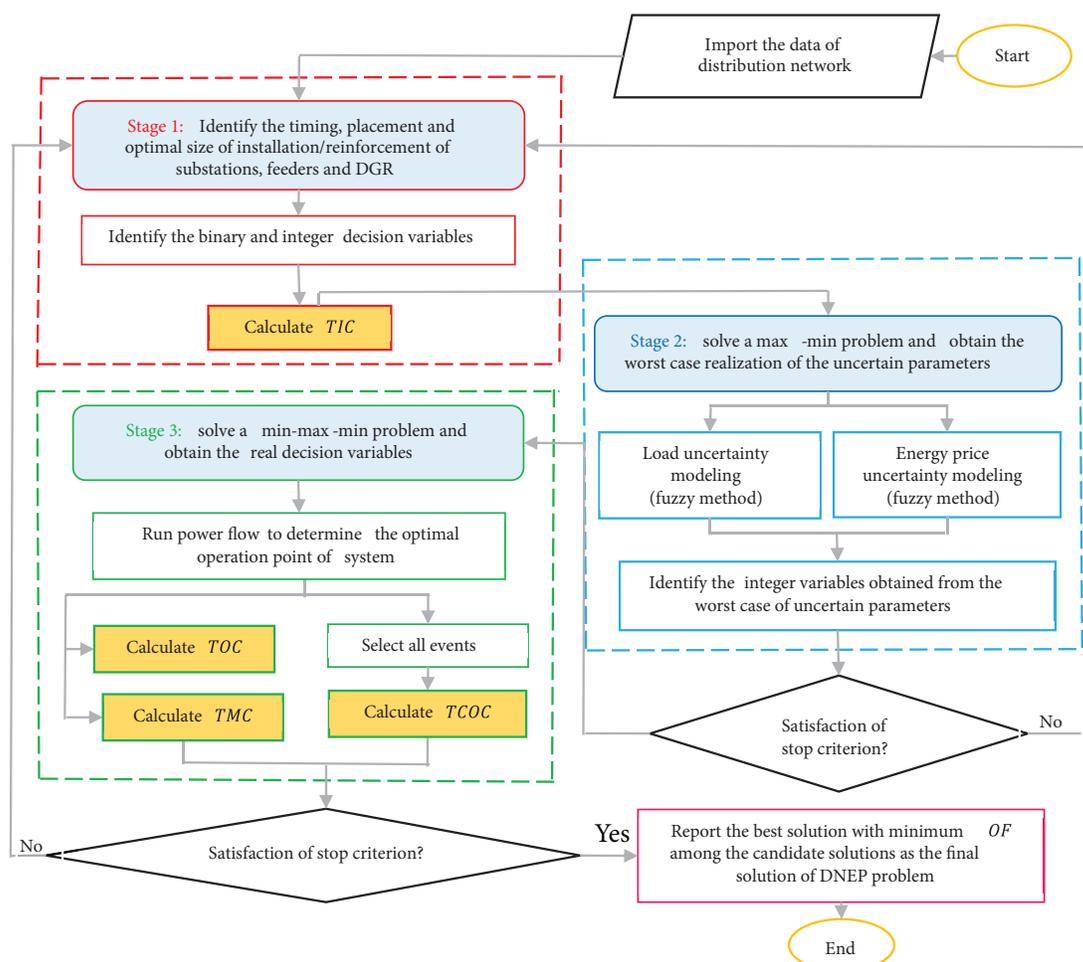


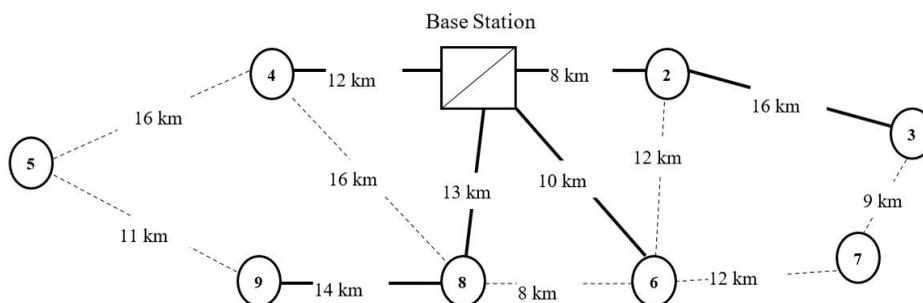
Figure 4. Flowchart of the proposed tri-stage decomposition approach for solving the DNEP problem.

A budget of uncertainty can be used to control the robustness of the proposed approach for characterization of the uncertain nature of loads and energy price. In this study,  $\Gamma$  is set to 8. By changing  $\Gamma$ , a trade-off

is established between the degree of confidence in the load values and objective function and satisfaction of the constraints.

#### 4. Numerical study

The DNEP is applied to a 33 kV distribution network to evaluate the proposed algorithm [28]. This network has a 40 MVA substation that can be reinforced to 80 MVA with six existing feeders and seven new feeders added to the installations. Figure 5 shows the primary distribution network and Table 1 lists sectors of the load points and annual peak power demand over the four years of the planning horizon. Solid lines in Figure 5 designate the existing feeders that can be reinforced, and dot lines are feeders with the capability of being added to the network.



**Figure 5.** Single-line diagram of the IEEE standard 9-bus system.

**Table 1.** Sectors and annual peak power demand of load points for 9-bus distribution system in the years of planning horizon.

Load point	Load sector	Peak demand (MVA)			
		Year 1	Year 2	Year 3	Year 4
2	Residential	5.3973	6.1860	6.6508	7.6400
3	Residential	4.4758	5.4800	6.7901	8.7200
4	Residential	5.3973	6.1860	6.6508	7.6400
5	Residential	—	—	3.4821	4.0000
6	Commercial	3.4891	3.7084	3.9870	4.5800
7	Commercial	—	4.4306	5.7455	7.2700
8	Industrial	4.6546	4.9472	5.3190	6.1100
9	Residential	3.6859	4.1618	4.4745	5.1400

Loading level, market price data, and cost of customer outage based on interruption duration are listed in Table 2. Techno-economical characteristics of the conductor types are presented in Table 3. Also, cost of the feeder reinforcement is set to 0.8 M\$/km, DGRs range between 1 MVA and 4 MVA in size, and all of the buses of the studied system are assumed as options for DGR installation. Other simulation parameters are given in Table 4. The average results for 10 independent runs are presented.

The proposed approach for the DNEP is applied in two different scenarios. In the first scenario, the purpose is to expand the network without DGRs, while DGRs are considered as expansion options in the second scenario. Now, according to Table 5, future load points are calculated based on the fuzzy theory for the last duration of programming, i.e. year 4.

**Table 2.** Network load levels, market energy price, and customer interruption data.

Load level	Percentage of peak load (%)	Time duration (h)	Market price (\$/MWh)	Load sector	Interruption duration				
					1 min	20 min	60 min	240 min	480 min
High	100	1500	70	Industrial	1.625	3.868	9.085	25.16	55.81
Normal	70	5000	49	Commercial	0.381	2.969	8.552	31.32	83.01
Low	50	2260	35	Residential	0.001	0.093	0.482	4.914	15.69

**Table 3.** Technical and economical characteristics of conductors used in the study.

Type	R ( $\Omega$ /km)	X X/km	Capacity (MVA)	Failure rate (failure/km year)	Repair time (h)	Cost (M\$/km)
1	0.1738	0.2819	12	0.096039	10.15	0.1
2	0.0695	0.2349	18	0.096039	10.15	0.15

**Table 4.** Technical/cost parameters of the study case network.

Parameter	Value
Discount rate (%)	12.5
Life-time maintenance cost for all equipment	3% of investment cost
Load power factor	0.85
DGR operation power factor	0.9
Allowed voltage deviation (%)	5
Cost of upgrading the substation (M\$)	0.8
DGR investment cost (M\$/MVA)	0.318
DGR operation cost (\$/MVA h)	50
Average switching time (h)	0.5

**Table 5.** Future load points obtained by fuzzy method for year 4.

Load point	Demand (MVA)		
	Minimum amount	Most likely amount	Maximum amount
2	6.876	7.6400	8.404
3	7.878	8.7200	9.592
4	6.876	7.6400	8.404
5	3.600	4.0000	4.400
6	4.122	4.5800	5.038
7	6.543	7.2700	7.997
8	5.499	6.1100	6.721
9	4.624	5.1400	4.454

Optimal expansion plans obtained from the proposed approach for the DNEP problem in scenarios 1 and 2 are given in Table 6. The terms "up1" and "up2" represent the installation/reinforcement of type 1 and type 2 feeders, respectively. Also, the terms "Ires1" and "Ires2" stand for the installation/reinforcement of type 1 and type 2 reserve feeders, respectively. Table 7 presents the cost components obtained for scenarios 1 and

2. The convergence trend of the GA-OPF process for the fourth year is depicted in Figure 6. For scenario 2, the effective solutions obtained by the proposed method are shown in Table 8. As can be observed, the 146th solution is the best.

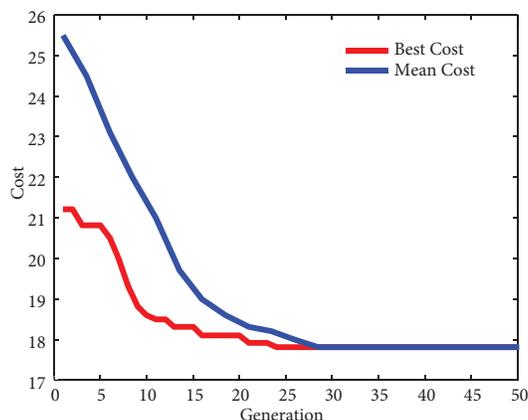
As can be observed, the application of DGRs reduces the cost of the DNEP. Also, due to the improvement of loading feeders, the costs of increasing feeder’s capacity are reduced. Simulation results indicated that eight feeders would be needed for 4 years of expansion planning in the absence of DGRs in the network. However, by considering DGRs, number of the new required feeders is reduced to 4. Also, by adding DGRs to the distribution network, there is no need for installing a new substation. Besides, TCOC is improved by the addition of DGRs to the distribution network. Figure 7 indicates the effects of DGRs on reliability. As can be seen in this figure, reliability is enhanced in all years by using DRGs in the network.

**Table 6.** Optimal expansion plans for 9-bus distribution system.

Planning horizon	Scenario 1		Scenario 2		
	Substation	Feeder	Substation	Feeders	DGR
First year	—	(6-8):Ires	—	(6-8):Ires	DGR(3):3MVA DGR(9):4 MVA
Second year	—	(1-2):up2 ,(1-8): up2 , (6-7): up1	—	(6-8): Ires 2, (6-7):up1	DGR(7):3MVA
Third year	Update	(1-4): up2, (2-3):up2, (4-5):up1, (9- 5):Ires	—	(2-3):up2, (4-5):up1, (9-5):Ires	DGR(5):4MVA
Fourth year	—	—	—	—	DGR(4):2MVA DGR(9):1MVA

**Table 7.** Cost components in the years of planning horizon for two scenarios.

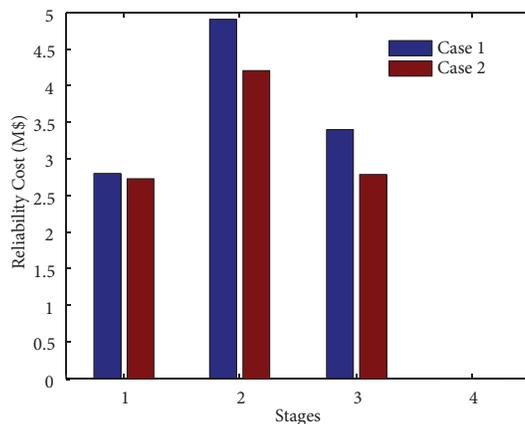
Cost Components	Scenario 1				Scenario 2			
	Year 1	Year 2	Year 3	Year 4	Year 1	Year 2	Year 3	Year 4
Feeder investment cost (M\$)	0.8	2.7750	4.8	0	0.8	1.2	3.9	0
Substation investment cost (M\$)	0	0	0.8	0	0	0	0	0
DGR investment cost (M\$)	0	0	0	0	2.226	0.954	1.727	0.954
TIC with CRF(M\$)	0.9271	3.2158	7.5203	0	2.7696	1.7591	5.6250	0.7370
TOC with CRF(M\$)	10.14	13.1341	22.2440	26.4807	8.2092	10.8755	17.3739	20.5178
TCOC with CRF(M\$)	2.5860	4.9429	3.3758	0	2.5105	4.2552	2.6761	0
Total cost (M\$)	13.6562	21.2927	33.1401	26.4807	15.7153	17.8438	27.402	22.2088



**Figure 6.** Convergence trend of the GA-OPF process for the second scenario.

**Table 8.** Effective fuzzy solutions under load and energy price uncertainties and the corresponding normalized value obtained by the proposed approach for the DNEP problem in the fourth year.

$\mu_k$	$f_{Enk}$	$f_{Tnk}$	k
0.0047	1.000	1.0000	3
0.2457	0.8714	0.9631	17
0.1562	0.9418	1.0000	39
0.0000	0.0721	0.0000	75
0.02790	0.2790	0.2965	105
0.4482	0.6544	0.9251	135
0.5412	0.5412	0.7753	146
0.3796	0.6313	0.9796	213
0.0413	0.4174	0.0413	263
0.3092	0.3092	0.4956	279
0.3696	0.3696	0.8328	329
0.3032	0.8167	1.0000	345
0.1940	0.1940	0.3119	346
0.2578	0.4505	0.2578	348
0.2648	0.5821	0.2648	370
0.2365	0.2365	0.7128	375
0.3990	0.6122	0.3990	434
0.4929	0.4929	0.5465	441



**Figure 7.** Comparison of customer outage cost during the years of planning horizon in the two scenarios.

### 5. Conclusion

The present paper introduced a model for multistage expansion planning in the distribution networks with DGRs. In this paper, to overcome the nonlinear mixed integer nature of the DNEP problem, a tri-stage robust model is proposed. The proposed model identifies timing of the installation/reinforcement of DGRs, feeders, and substations, as well as their locations and capacities. Also, a techno-economic approach has been proposed to minimize the total costs and improve the reliability of the network. A hybrid GA-OPF algorithm is implemented to solve the optimization problem.

Power demands and energy price can deviate from their predicted values in practice, where there is no guarantee of an optimal or even a feasible solution of a deterministic tool. In this paper, the fuzzy theory has been considered to obtain a robust solution against different realizations of uncertain parameters.

The obtained results have illustrated that:

1. Applying DGR would be effective in reducing the total costs.
2. Expanding range of the uncertainty deviation yields a more robust and flexible solution, however, with a higher total cost.
3. At the expense of a higher computation burden, uncertainty characterization via a higher number of operating conditions has diminished the total costs. The tri-stage decomposition approach, proposed to solve this model, significantly reduced the computational burden of the problem. Also, the division of a large-scale MINLP optimization problem into MILP and NLP subproblems with smaller sizes made the optimization problem more tractable and decrease the computation time.

The proposed approach in the DNEP problem can be enhanced by numerous methods, such as using a broad range of expansion options, improvement of model robustness under various uncertainties, applying adaptive models to address more uncertainty sources, expansion planning in presence of energy storage systems, that can be investigated in the future works.

<b>Nomenclature</b>				
Indices				
$y$	Index of expansion planning	$N_d$	Number of DGRs	
$p$	Period index of expansion horizon	$N_f$	Number of feeders	
$s$	Index of substation	$N_s$	Number of substations	
$l$	Index of load level	$N_l$	Number of load levels	
$f$	Index of feeder	$N_i$	Number of load points	
$d$	Index of DGR	$N_p$	Number of periods in expansion horizon	
$m$	Index of load sector	$I_{s,p}^{oper}$	Injected current by substation $s$ during period $p$ [p.u]	
$i$	Index of load point	$\rho_{i,m}^e$	Average failure rate affected load point $i$ at load sector $m$ and event $e$	
$e$	Failure event index	$TB_p$	Total budget during period $p$	

Parameters		$Z_f$	Impedance of feeder $f$
$C_{y,p}^{inv}$	Investment cost of plan $y$ during period $p$ [M\$]	$Y_i^{ins}$	Installation year of feeder/DGR in bus $i$
$C_{s,p}^{rein}$		$\Gamma$	Budget of uncertainty
$C_{f,p}^{ins}$	Installation cost of feeder $f$ during period $p$ [M\$/km]	$I_{f,p}^{oper}$	Current flowing through feeder $f$ during period $p$ [p.u.]
$C_{f,p}^{rein}$	Reinforcement cost of feeder $f$ during period $p$ [M\$/km]	$P_{d,l,p}^{oper}$	Generated power of DGR $d$ in load level $l$ during period $p$ [MW]
$C_{s,p}^{rein}$	Reinforcement cost of substation $s$ during period $p$ [M\$/km]	$\gamma_{s,l,p}^{oper}$	Electrical energy price of substation $s$ at load level $l$ during period $p$ [\$/MWh]
$C_{d,p}^{ins}$	Installation cost of DGR $d$ during period $p$ [M\$/MVA]	$P_{s,l,p}^{oper}$	Injected power from substation $s$ at load level $l$ during period $p$ [MW]
$\sigma_{d,l,p}^{oper}$	Operational cost of DGR $d$ at load level $l$ during period $p$ [\$/MWh]	$\beta_f^{rein}$	A binary decision variable representing the reinforcement status of feeder $f$
$C_{y,p}^{maint}$	Maintenance cost of plan $y$ during period $p$ [M\$]	$\beta_s^{rein}$	A binary decision variable representing the reinforcement status of substation $s$
$C_{y,s,p}^{maint}$	Maintenance cost of substation $s$ in plan $y$ during period $p$ [M\$]	$C_{y,l,p}^{cu-out}$	Customers outage cost of plan $y$ in failure event $e$ and at load level $l$ during period $p$ [M\$]
$C_{y,f,p}^{maint}$	Maintenance cost of feeder $f$ in plan $y$ during period $p$ [M\$]	$P_{i,m,l,p}^e$	Real power demand of the load point $i$ in load sector $m$ , event $e$ , and at load level $l$ during period $p$ [MW]
$Ir$	Interest rate	$\hat{u}$	Forecast of $U$
$\delta$	Lifetime of plan $y$	$\hat{U}$	Deviation of $U$
$\beta_d^{ins}$	A binary decision variable representing the installation status of DGR $d$	$S_I$	Objective function of the first-stage problem
$\beta_f^{ins}$	A binary decision variable representing the installation status of feeder $f$	$S_{II}$	Objective function of the second-stage problem
$Y_i^{rein}$	Reinforcement year of substation/feeder in bus $i$	$S_{III}$	Objective function of the third-stage problem
$IM_{row,f}^\gamma$	Row $f$ in the transpose matrix of intersection branches and nodes	$f_{i,m,l,p}(r_{i,e})$	The per-unit cost of an outage based on the outage time $r_i$ in load sector $m$ , event $e$ , and at load level $l$ during period $p$
$TOC_y$	Operational cost of plan $y$ [M\$]	$EMC_l$	Energy price at load level $l$ [\$/MWh]
$TMC_y$	Maintenance cost of plan $y$ [M\$]	$r_{i,m,e}$	Average restoration time affected by bus $i$ in load sector $m$ and event $e$
$TCOC_y$	Customers outage cost of plan $y$ [M\$]	$USF(x)$	Unit step function, where $USF(x) = 1$ if $x > 0$ ; else, $USF(x) = 0$
$V_{i,l,p}^{max}$	Upper limit of voltage in bus $i$ and load level $l$ during period $p$	$C_{y,l,p}^{oper}$	Operational cost of plan $y$ at load level $l$ during period $p$ [M\$]
$V_{i,l,p}^{min}$	Lower limit of voltage in bus $i$ and load level $l$ during period $p$	Set	
$P_d^{min}$	Lower limit of active power for DGR [kW]	$\mu^{IS}$	Set of investment constraints
$P_d^{max}$	Upper limit of active power for DGR [kW]	$\mu^{US}$	Uncertainty set
$Q_d^{min}$	Lower limit of reactive power for DGR [kVAR]	$\sigma^{AC-OPF}$	Set of the non-linear AC-OPF constraints
$Q_d^{max}$	Upper limit of reactive power for DGR [kVAR]	$E$	Set of all failure events
$Q_{d,p}$	Output reactive power of DGR $d$ during period $p$ [kVAR]	Vector	
$LLD_l$	Time duration at load level $l$ [hr]	$I_{l,p}$	Feeder current vector at load level $l$ during period $p$
$I_{f,p}^{max}$	Upper limit of the current flowing through feeder $f$ during period $p$ [p.u.]	$EDC_{l,p}$	Interrupted bus vector at load level $l$ during period $p$
$I_{s,p}^{max}$	Upper limit of injected current by substation $s$ during period $p$ [p.u.]	$ED_{l,p}$	Electric power demand vector for node at load level $l$ during period $p$
$P_d^{cap}$	Operational capacity of DRG $d$ [MVA]	$U$	Vector of uncertain parameters
$P_d^{res}$	Reserve capacity of DRG $d$ [MVA]	$\alpha, \beta$	Vectors of investment and operation costs
$TIC_y$	Investment cost of plan $y$ [M\$]	$R, P$	Vectors of investment and operation variables
$V_{i,l,p}$	Voltage in bus $i$ at load level $l$ during period $p$	$I(R, P, U)$	Non-linear function vector
$R_s$	Resistance of substation $s$	$A$	Matrix of coefficients
$FL_f$	Length of feeder $f$ [km]	$B$	Vector of requirements
$P_{i,p}$	Load demand in bus $i$ during period $p$ [kW]	$IM^\gamma$	Transpose matrix of intersection branches and nodes
$R_f$	Resistance of feeder $f$	$IN_{l,p}$	Bus injection vector at load level $l$ during period $p$

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